Binary Search Trees (BST)

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Review: Linear Search

- Collection of data items to be searched is organized in a list

  $x_1, x_2, \ldots x_n$

  – Assume == and < operators defined for the type
- Linear search begins with item 1
  – continue through the list until target found
  – *or* reach end of list

Linear Search

Array based search function

```
void LinearSearch (int v[], int n,
                   const int& item,
                   boolean &found, int &loc)
{
  found = false;  loc = 0;
  for ( ; ; )
  {
    if (found || loc < n) return;
    if (item == v[loc])  found = true;
    else loc++;
  }
}
```

Linear Search

Singly-linked list based search function

```
void LinearSearch (NodePointer first,
          const int& item, bool &found, int
   &loc)
{
   NodePointer locptr=first;
   found = false;
   for{loc=0; !found && locptr!=NULL;
      locptr=locptr->next)
   {
      if (item == locptr->data)
           found = true;
      else loc++;
   }
}
```

Binary Search

- Two requirements?

Binary Search

- Two requirements
  - The data items are in ascending order (can they be in decreasing order?)
  - Direct access of each data item for efficiency (why linked-list is not good!)

Binary Search

Binary search function for vector

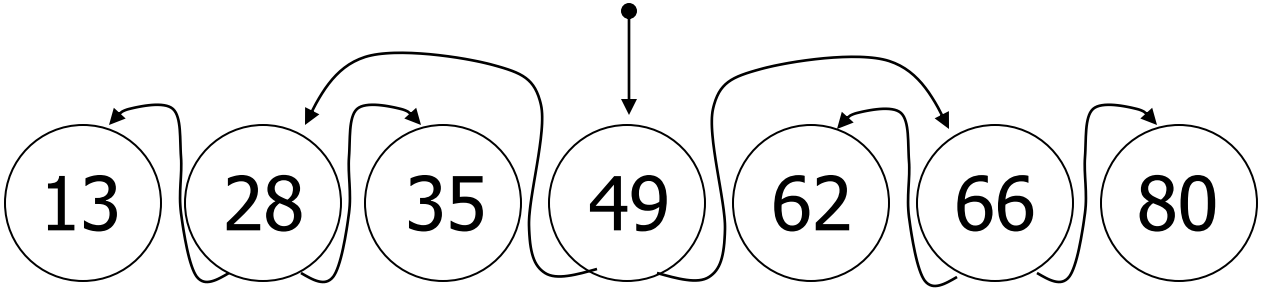
```
void LinearSearch (int v[], int n,
                   const t &item,
                   bool &found, int &loc) {
  found = false; loc = 0;
  int first = 0;   int last = n - 1;
  while (first <= last){
    loc = (first + last) / 2;
    if (item < v[loc])
        last = loc - 1;
    else if (item > v[loc])
        first = loc + 1;
    else {
        found = true; // item found
        break;
    }
  }
}
```

Binary Search vs. Linear Search

- Usually outperforms Linear search: O(logn) vs. O(n)
- Disadvantages
  - Sorted list of data items
  - Direct access of storage structure, not good for linked-list
- Good news: It is possible to use a linked structure which can be searched in a binary-like manner

Binary Search Tree (BST)

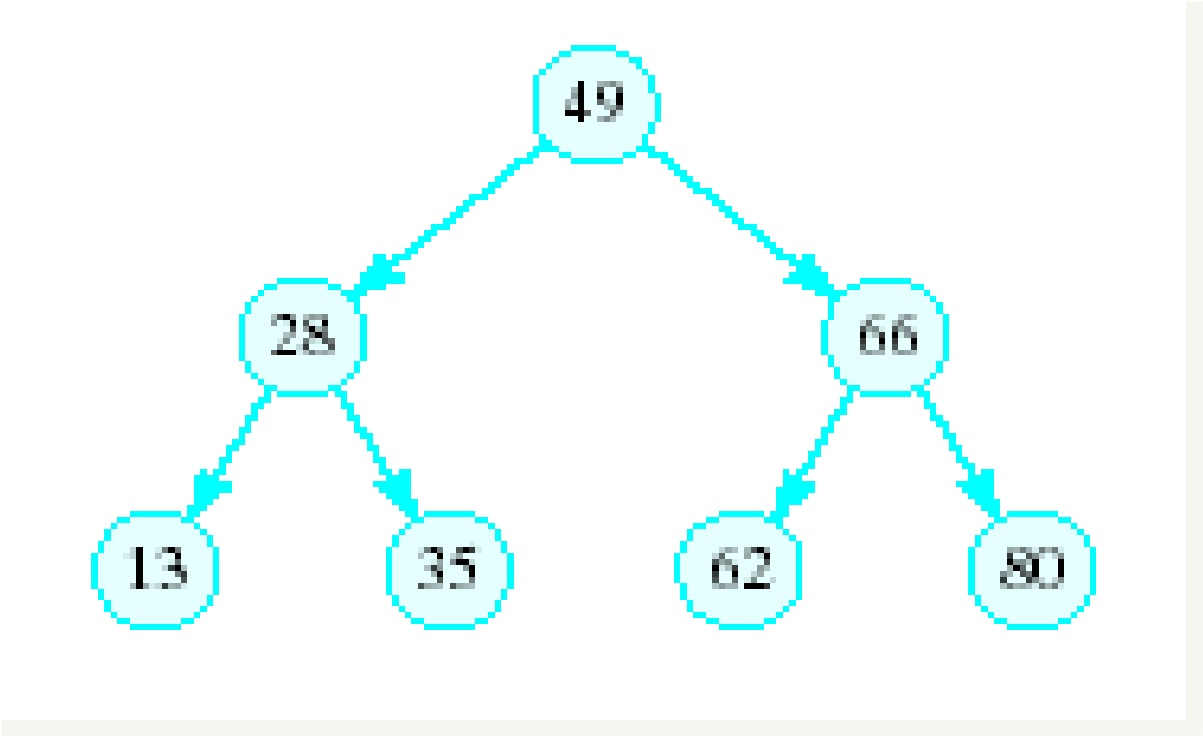
- Consider the following ordered list of integers



1. Examine middle element
2. Examine left, right sublist (maintain pointers)
3. (Recursively) examine left, right sublists

Binary Search Tree

- Redraw the previous structure so that it has a treelike shape – a binary tree

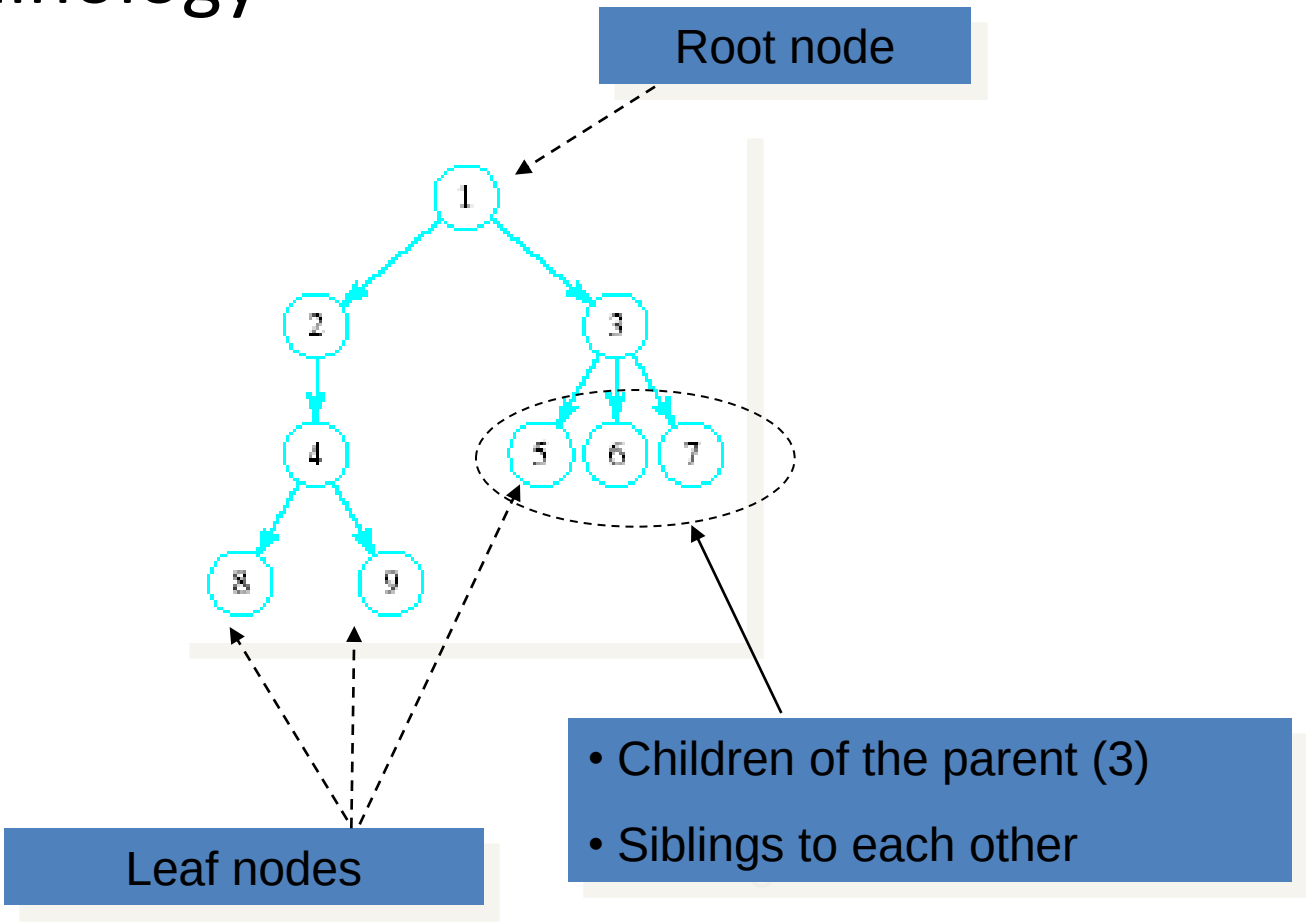


Trees

- A data structure which consists of
  - a finite set of elements called nodes or vertices
  - a finite set of directed arcs which connect the nodes
- If the tree is nonempty
  - one of the nodes (the root) has no incoming arc
  - every other node can be reached by following a unique sequence of consecutive arcs (or paths)

Trees

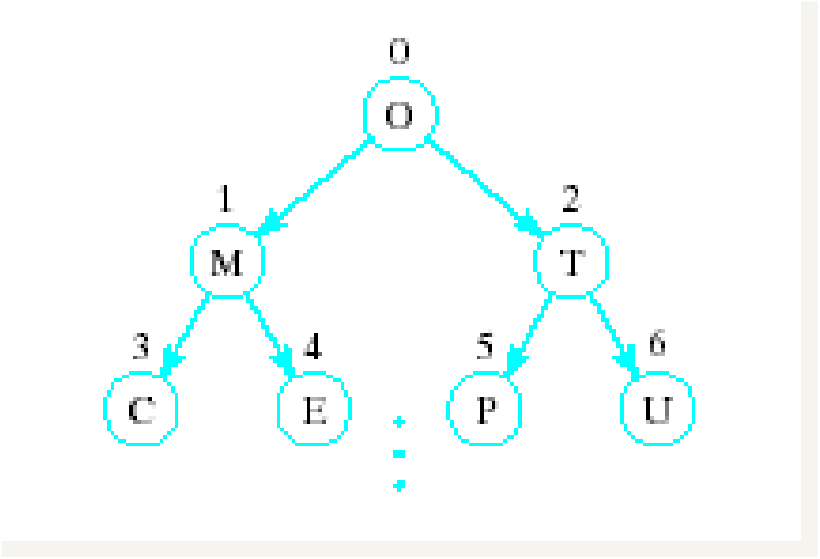
- Tree terminology



Binary Trees

- Each node has at most two children
- An empty tree is a binary tree

Array Representation of Binary Trees

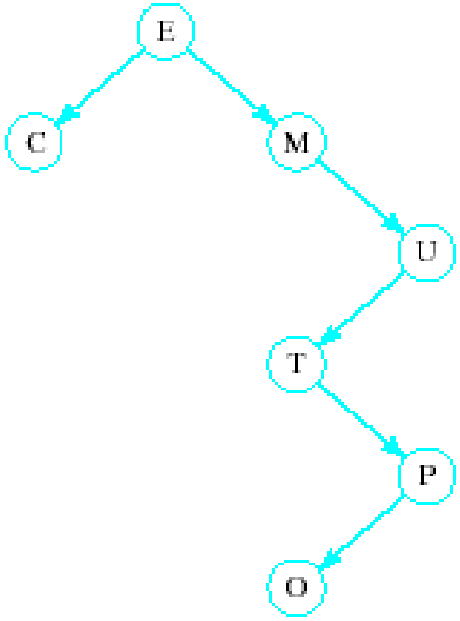


i	0	1	2	3	4	5	6	...
$t[i]$	O	M	T	C	E	P	U	...

- Store the i^{th} node in the i^{th} location of the array

Array Representation of Binary Trees

- Works OK for complete trees, not for sparse trees



Some Tree Definition, p656

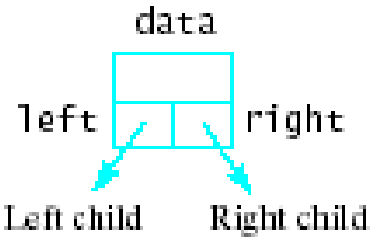
- Complete trees (might different form other books)
  - Each level is completely filled except the bottom level
  - The leftmost positions are filled at the bottom level
  - Array storage is perfect for them
- Balanced trees
  - Binary trees
  - |left_subtree – right_subtree|<=1
- Tree Height/Depth:
  - The # of levels

Tree Questions

- A complete tree must be a balanced tree?
- Give a node with position *i* in a complete tree, what are the positions of its child nodes?
  - Left child?
  - Right child?

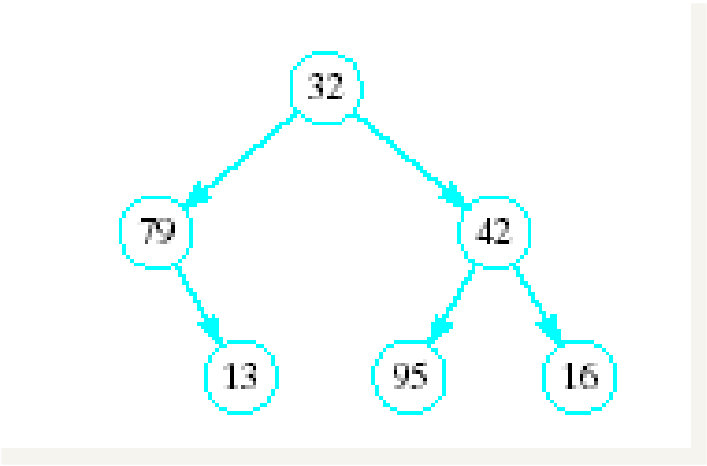
Linked Representation of Binary Trees

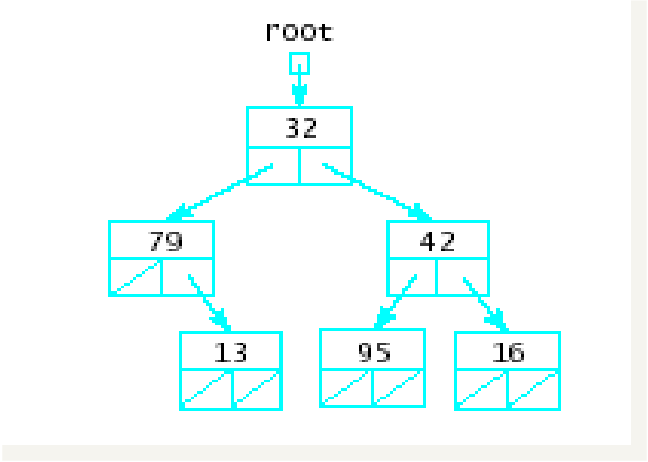
- Uses space more efficiently
- Provides additional flexibility
- Each node has two links
  - one to the left child of the node
  - one to the right child of the node
  - if no child node exists for a node, the link is set to NULL



Linked Representation of Binary Trees

- Example





Binary Trees as Recursive Data Structures

- A binary tree is either empty …

  or

- Consists of
  - a node called the root
  - root has pointers to two disjoint binary (sub)trees called …
    - right (sub)tree
    - left (sub)tree

Anchor

Inductive step

Which is either empty … or …

Which is either empty … or …

Tree Traversal is Recursive

If the binary tree is empty then
do nothing

Else

N: Visit the root, process data
L: Traverse the left subtree
R: Traverse the right subtree

The "anchor"

The inductive step

Traversal Order

Three possibilities for inductive step …

- Left subtree, Node, Right subtree
  the inorder traversal

- Node, Left subtree, Right subtree
  the preorder traversal

- Left subtree, Right subtree, Node
  the postorder traversal

ADT: Binary Search Tree (BST)

- Collection of Data Elements
  - Binary tree
  - BST property: for each node x,
    - value in left child of x < value in x < in right child of x
- Basic operations
  - Construct an empty BST
  - Determine if BST is empty
  - Search BST for given item

ADT Binary Search Tree (BST)

- Basic operations (ctd)
  - Insert a new item in the BST
    - Maintain the BST property
  - Delete an item from the BST
    - Maintain the BST property
  - Traverse the BST
    - Visit each node exactly once
    - The *inorder traversal* must visit the values in the nodes in ascending order

BST

```
typedef   int   T;
class BST {
    public:
       BST() : myRoot(0) {}
       bool empty() { return myRoot==NULL; }
    private:
       class BinNode {
         public:
            T   data;
            BinNode* left, right;
            BinNode() : left(0), right(0) {}
            BinNode(T item): data(item), left(0), right(0) {}
       }; //end of BinNode
       typedef BinNode*   BinNodePtr;
       BinNodePtr  myRoot;
};
```

BST operations

- Inorder, preorder and postorder traversals (recursive)
- Non-recursive traversals

BST Traversals, p.670

- Why do we need two functions?
  - Public: void inorder(ostream& out)
  - Private: inorderAux(…)
    - Do the actual job
    - To the left subtree and then right subtree
- Can we just use one function?
  - Probably NO

Non-recursive traversals

- Let’s try non-recusive inorder
- Any idea to implement non-recursive traversals? What ADT can be used as an aid tool?

Non-recursive traversals

- Basic idea
  - Use LIFO data structure: stack
  - #include <stack> (provided by STL)
  - Chapter 9, google stack in C++ for more details, members and functions
  - Useful in your advanced programming

Non-recursive inorder

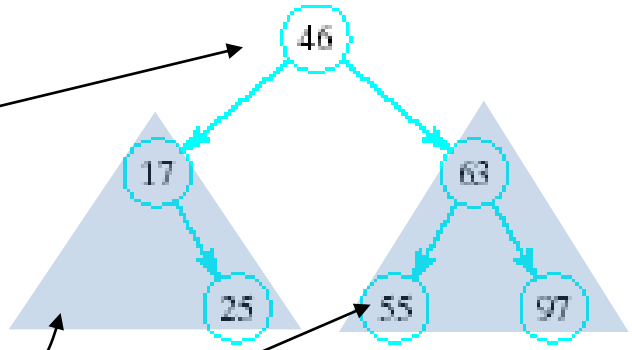
- Let’s see how it works given a tree
- Step by step

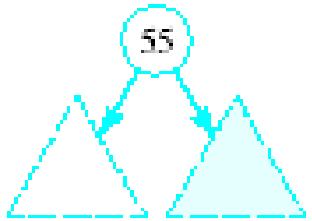
Non-recursive traversals

- Can you do it yourself?
  - Preorder?
  - Postorder?

BST Searches

- Search begins at root
  - If that is desired item, done
- If item is less, move down left subtree
- If item searched for is greater, move down right subtree
- If item is not found, we will run into an empty subtree





BST Searches

- Write recursive search
- Write Non-recursive search algorithm
- bool search(const T& item) const

Insertion Operation

Basic idea:

- Use search operation to locate the insertion position or already existing item
- Use a parent point pointing to the parent of the node currently being examined as descending the tree

Insertion Operation

- Non-recursive insertion
- Recursive insertion
- Go through insertion illustration p.677
  - Initially *parent = NULL, locptr = root;*
  - Location termination at NULL pointer or encountering the item
  - Parent->left/right = item

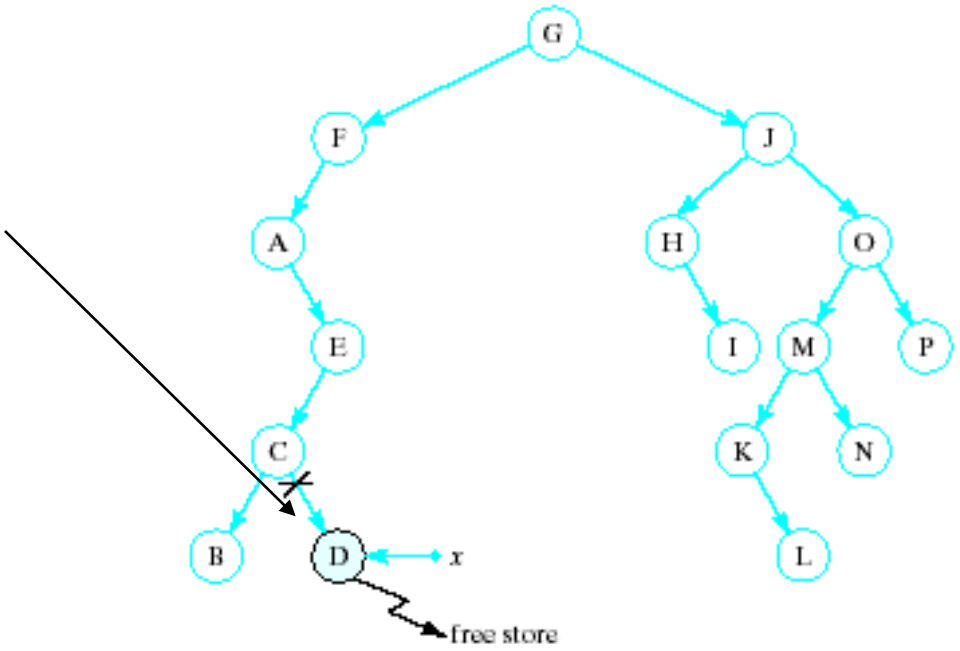
Recursive Insertion

- See p. 681
- Thinking!
  - Why using & at *subtreeroot*

Deletion Operation

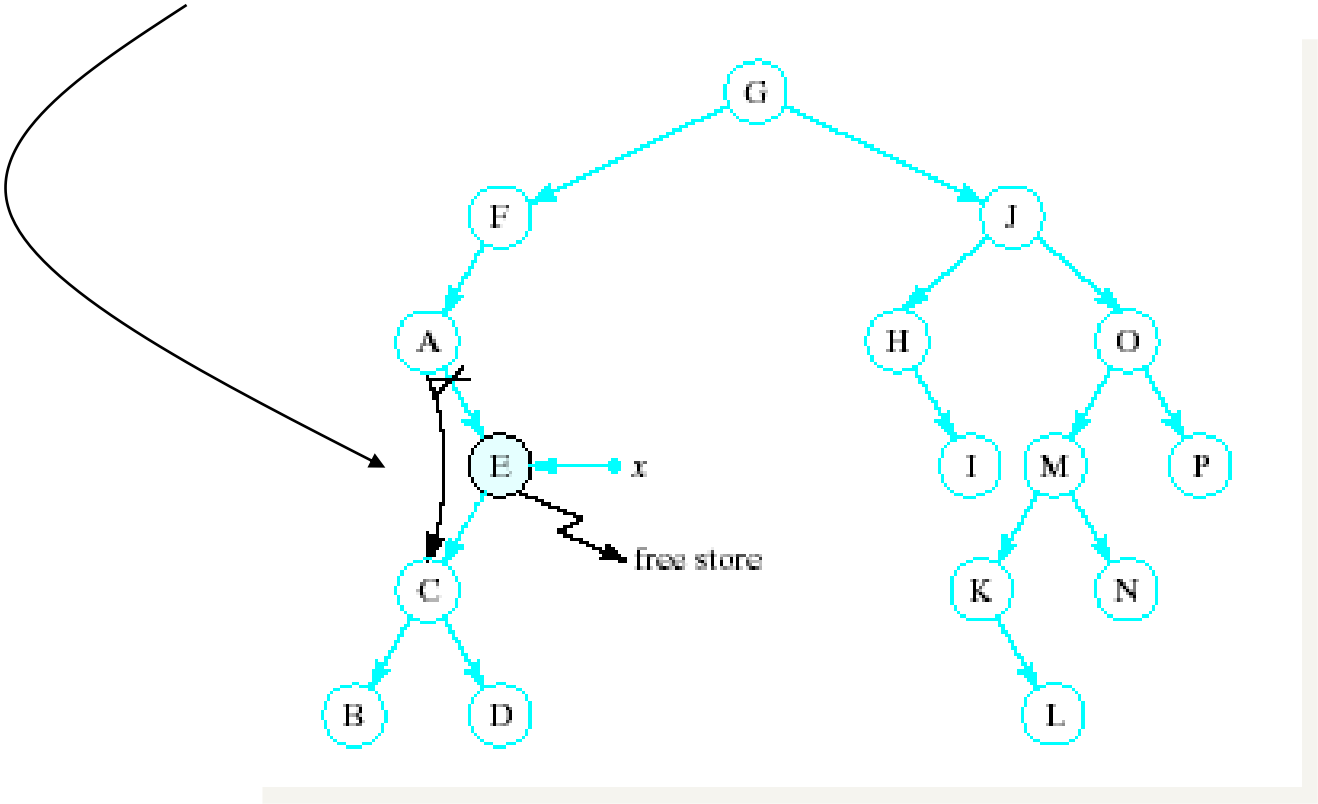
Three possible cases to delete a node, *x*, from a BST

1. The node, *x*, is a leaf



Deletion Operation

2. The node, *x* has one child

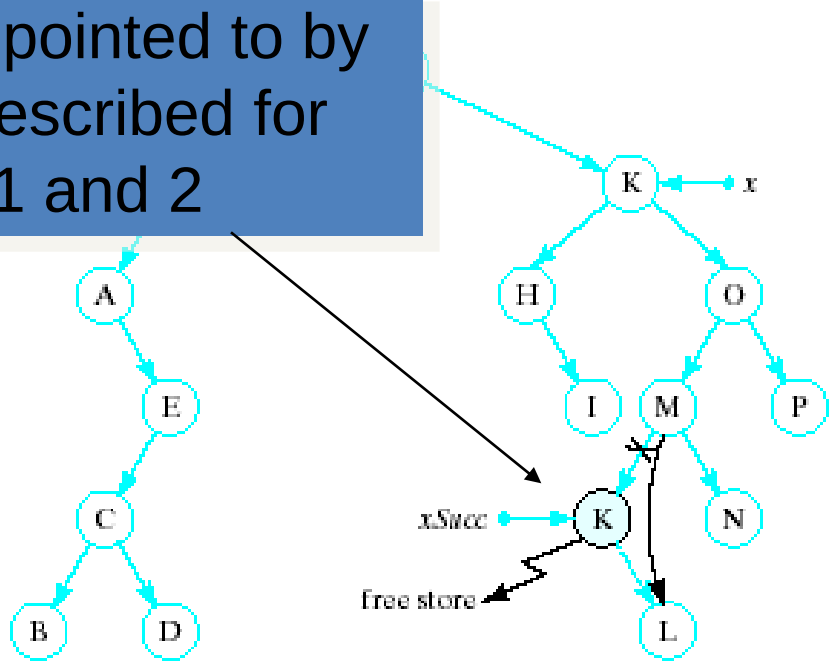


Deletion Operation

- *x* has two children

Delete node pointed to by xSucc as described for cases 1 and 2

Replace conten
inorder succ



View **`remove()`** function

Question?

- Why do we choose inorder predecessor/successor to replace the node to be deleted in case 3?

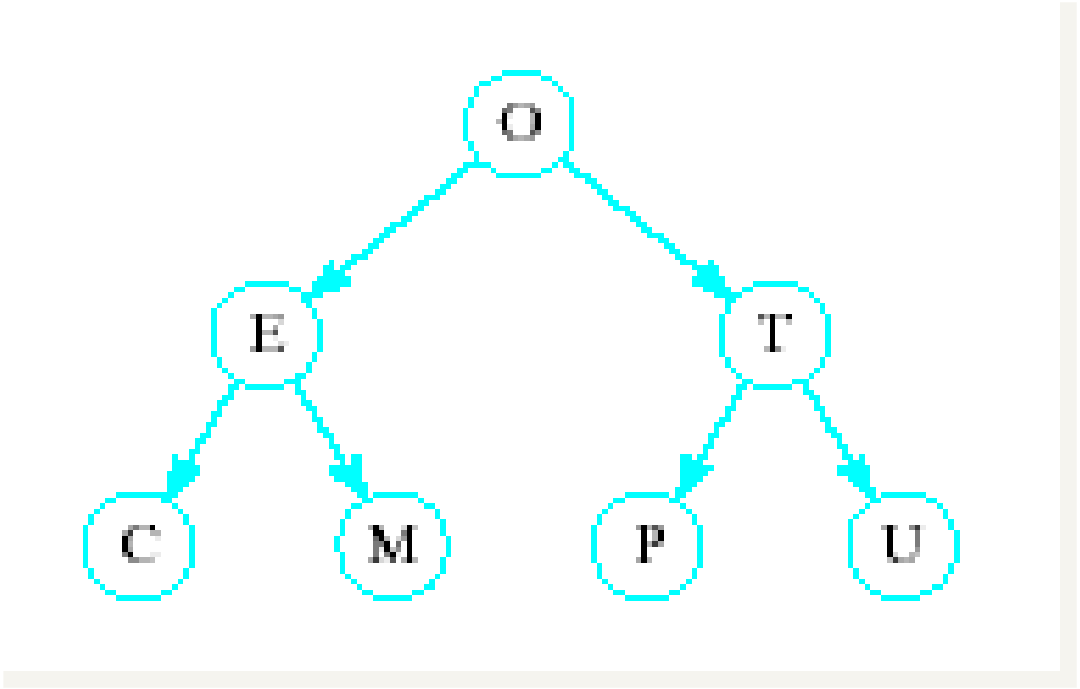
Questions?

- What is T(n) of search algorithm in a BST? Why?
- What is T(n) of insert algorithm in a BST?
- Other operations?

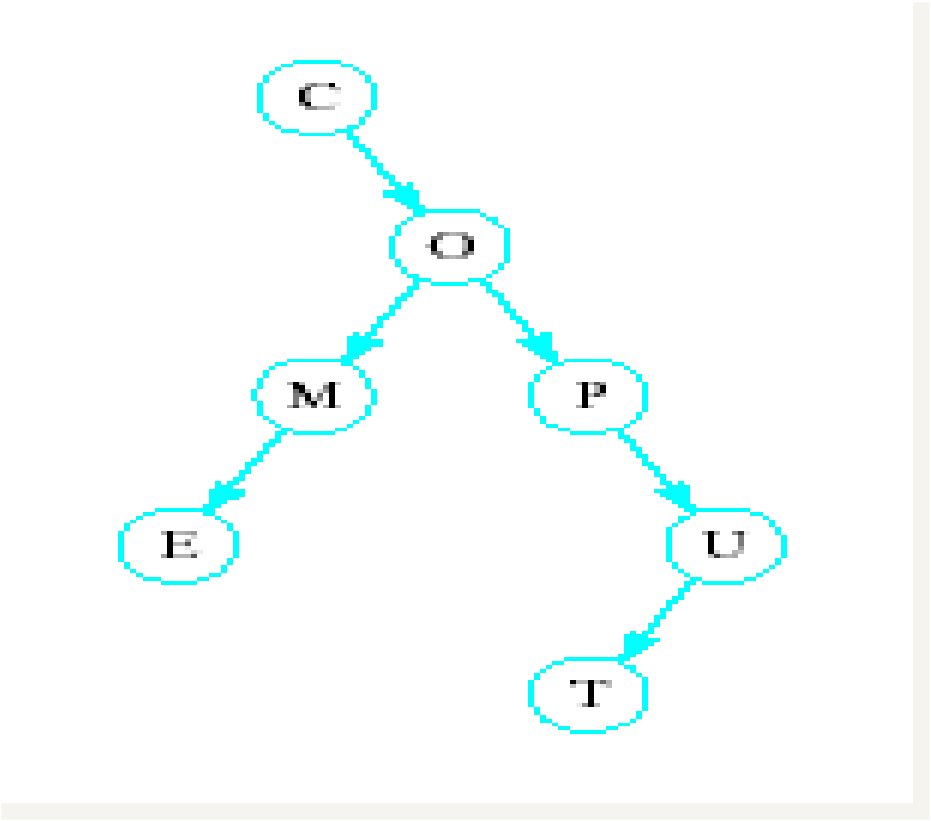
Problem of Lopsidedness

- The order in which items are inserted into a BST determines the shape of the BST
- Result in Balanced or Unbalanced trees
- Insert O, E, T, C, U, M, P
- Insert C, O, M, P, U, T, E

Balanced

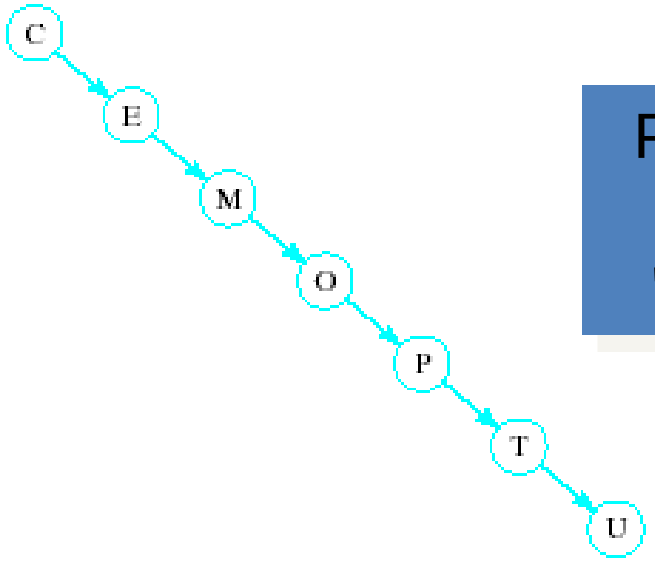


Unbalanced



Problem of Lopsidedness

- Trees can be totally lopsided
  - Suppose each node has a right child only
  - Degenerates into a linked list



Processing time affected by "shape" of tree