Tree Balancing: AVL Trees

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Recall in BST

- The insertion order of items determine the shape of BST
 - Balanced: search T(n)=O(logN)
 - Unbalanced: T(n) = O(n)
- Key issue:
 - Need to keep a BST balanced!
 - Tree balancing techniques

AVL Tree Definition

- First, a BST
- Second, height-balance property: balance factor of each node is 0, 1, or -1
- Question: what is balance factor?

AVL Tree Definition

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BF = Height of the left subtree – height of the right subtree Height: # of levels in a subtree/tree

Determine balance factor



ADT: AVL Trees

• Data structure to implement



ADT: AVL Trees

- Basic operations
 - Constructor, search, traversal, empty
 - Insert: keep balanced!
 - Delete: keep balanced!
 - See P842 class declaration
 - Similar to BST

Example



Insert "DE", what happens? Need rebalancing?

Basic Rebalancing Rotation

- Single rotation:
 - Right rotation: the inserted item is on the Left subtree of Left child of the nearest ancestor with BF of 2
 - Left rotation: the inserted item is on the Right subtree of Right child of the nearest ancestor with BF of -2
- Double rotation
 - Left-right rotation: the inserted item is on the Right subtree of Left child of the nearest ancestor with BF of 2
 - Right-left rotation: the inserted item is on the Left subtree of Right child of the nearest ancestor with BF of -2

How to perform rotations

• Rotations are carried out by resetting links

- Two steps:
 - Determine which rotation
 - Perform the rotation

Right Rotation

- Key: identify the nearest ancestor of inserted item with BF +2
- A: the nearest ancestor.
- **B**: left child
- How? (clockwise rotation)
 - Step1: reset the link from parent of A to B (promote B)
 - Step2: set the left link of A equal to the right link of B
 - Step3: set the right link of B to A (demote A)





How about insert



Left Rotation

- How? (counter-clockwise rotation)
 - Step1: reset the link from parent of A to B (promote B)
 - Step2: set the right link of A to the left link of B
 - Step3: set the left link of B to A (demote A)









How about insert



Double Rotation

- Left-right rotation: the inserted item is on the Right subtree of Left child of the nearest ancestor with BF of 2
- Right-left rotation: the inserted item is on the Left subtree of Right child of the nearest ancestor with BF of -2

Double Rotations

- How to perform?
 - Step 1: Rotate child and grandchild nodes of the ancestor (grandchild < -- > child)
 - Step 2: Rotate the ancestor and its new child node

Example









How about insert







Summary on Rebalancing

- AVL tree definition
- The key is you need to identify the nearest ancestor of inserted item
- Determine the rotation by definition
- Perform the rotation
- Implementation of AVL trees is not required in this class