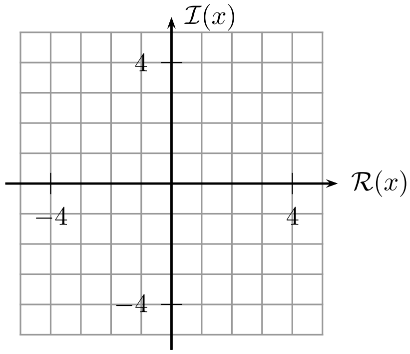
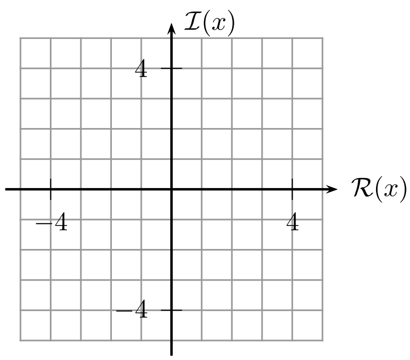
MATH 2340 RADIUS PRACTICE

Consider the following functions $f(x)$. Use complex variables to find the radius of convergence for the Taylor Series centered about the given x_0. Then list the real values of x for which the series will converge.

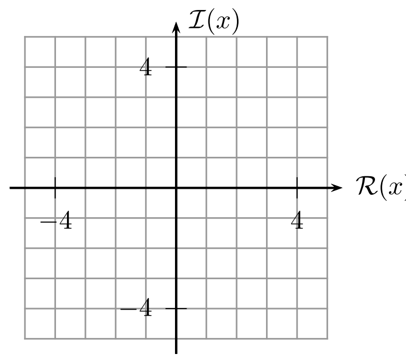
1. $f(x) = \dfrac{1}{x^2 - 1}$, where $x_0 = 2$



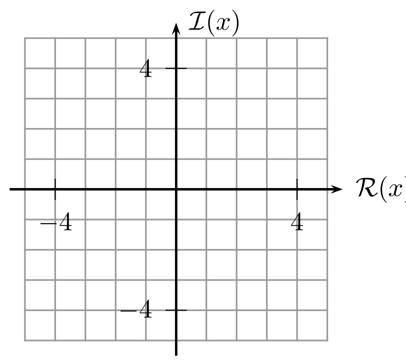
2. $f(x) = \dfrac{x}{x^2 + 2x + 2}$, where $x_0 = \frac{1}{2}$



3. $f(x) = \dfrac{x}{x^2 - x}$, where $x_0 = 3$



4. $f(x) = \dfrac{7x - 1}{x^3 + x}$, where $x_0 = 2$



MATH 2340 INFO ABOUT THE RADIUS OF CONVERGENCE

THEOREM

If x_0 is an ordinary point of the differential equation

$$P(x)y'' + Q(x)y' + R(x)y = 0, \tag{1}$$

that is, $p(x) = Q(x)/P(x)$ and $q(x) = R(x)/P(x)$ are analytic at $x = x_0$, then the general solution of (1) is given by

$$y(x) = \sum_{n=0}^{\infty} a_n (x - x_0)^n = a_0 y_1 + a_1 y_2$$

where a_0 and a_1 are arbitrary constants, and y_1 and y_2 are linearly independent series solutions. Furthermore, $y(x)$ is analytic at $x = x_0$, and the radius of convergence for $y(x)$ is at least as large as the radius of convergence for the series expansion of $p(x)$ and $q(x)$.

In order to find information about the radius of convergence for a series solution to a differential equation, there are three options:

1. Find the series solution $y(x)$ and then test for convergence.
2. Find the series expansion for $p(x)$ and $q(x)$. Find the x values for which it converges using a convergence test. This gives a lower bound on the radius of convergence for $y(x)$.
3. Use complex variables to find where the series for $p(x)$ and $q(x)$ converge. This idea can be summarized as follows: *If $f(x)$ is a rational function, then the series expansion for $f(x)$ about a point x_0 has a radius of convergence that is precisely the distance from x_0 to the closest singularity of $f(x)$ (think zero in the denominator).*

Here are some facts you might need:

- $\sum_{n=0}^{\infty} b_n$ converges absolutely if
$$\lim_{n\to\infty} \left|\frac{b_{n+1}}{b_n}\right| < 1$$
- If $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$ are complex numbers, then the distance between z_1 and z_2 is given by
$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$