Math 2340 Warm-Up Problems

1. Consider the second-order differential equation

$$y'' + p(x)y' + q(x)y = g(x). \tag{1}$$

Under what special circumstance is (1) a *homogeneous* ODE? What about *linear*?

2. Now, consider the initial value problem

$$y'' + p(x)y' + q(x)y = 0, \qquad y(0) = b_1, \quad y'(0) = b_2.$$

Assume that the general solution to the above differential equation is given by $y(x) = c_1 y_1(x) + c_2 y_2(x)$ where both $y_1(x)$ and $y_2(x)$ both solve the ODE and c_1 and c_2 are arbitrary constants.

Write a system of two equations involving the initial conditions that allows you to solve for the constants c_1 and c_2. If possible, write your equations in matrix form $\mathbf{A}\,\vec{\mathbf{c}} = \vec{\mathbf{b}}$ where A is a 2×2 matrix, and $\vec{\mathbf{b}}$ and $\vec{\mathbf{c}}$ are 2×1 vectors.

3. What must be true about the expression derived above in order to solve for c_1 and c_2 given any set of initial conditions (any value for b_1 and b_2)?

That's it. No more fun on the other side.