MATH 2340 "COOL DOWN" PROBLEMS

For each of the following differential equations, (1) determine the fundamental solutions, and (2) write down the general solution.

(1) $y'' + 2y' - 3y = 0$

Assume $y = e^{\lambda x} \rightarrow y' = \lambda e^{\lambda x} \rightarrow y'' = \lambda^2 e^{\lambda x}$

Plug in

$$\lambda^2 e^{\lambda x} + 2\lambda e^{\lambda x} - 3e^{\lambda x} = 0$$

$$e^{\lambda x}(\lambda^2 + 2\lambda - 3) = 0 \quad \text{Note: } e^{\lambda x} \neq 0$$

$$\lambda^2 + 2\lambda - 3 = 0 \leftarrow \text{characteristic equation}$$

$$(\lambda + 3)(\lambda - 1) = 0$$

$$\lambda = -3 \text{ or } \lambda = 1 \qquad y_1 = e^{-3x},\ y_2 = e^{x}$$

Write solution

$$\boxed{y = c_1 e^{-3x} + c_2 e^{x}}$$

(2) $y'' - 9y = 0$

Assume $y = e^{\lambda x}$

plug in

$$\lambda^2 - 9 = 0 \leftarrow \text{characteristic equation}$$

$$\lambda = \pm 3$$

Write solution

$$\boxed{y = c_1 e^{3x} + c_2 e^{-3x}}$$

(3) $y'' - 7y' = 0$

Assume $y = e^{\lambda x}$

Plug in

$$\lambda^2 - 7\lambda = 0$$

$$\lambda(\lambda - 7) = 0$$

$$\lambda = 0 \text{ or } \lambda = 7 \qquad y_1 = e^{0x},\ y_2 = e^{7x}$$

Write Solution

$$y = c_1 + c_2 e^{7x}$$

(4) $y'' = 0$ Assume $y = e^{\lambda x}$

Plug in $\lambda^2 = 0 \quad \lambda = 0$? only 1?

Try a different route.

$$y'' = 0 \rightarrow \int y'' dx = \int 0\, dx$$

$$y' = c_1$$

$$\int y' dx = \int c_1 dx$$

$$\boxed{y = c_1 x + c_2}$$