MATH 2340 WARM-UP PROBLEMS

1. Assuming that $F(s)$ is the Laplace transform of $f(t)$ ($\mathcal{L}\{f(t)\} = F(s)$), determine the appropriate values for the constants a, b, and c.

$$f(t) = at - \frac{1}{8}\sin(2t)$$

$$F(s) = \frac{1}{s^b\,(s^2+c)}$$

Taking the Laplace transform of $f(t)$, we find:

$$\mathcal{L}\{f(t)\} = a\mathcal{L}\{t\} - \tfrac{1}{8}\mathcal{L}\{\sin(2t)\}$$

using the LT table.

$$F(s) = a\cdot\frac{1}{s^2} - \frac{1}{8}\cdot\frac{2}{s^2+4}$$

Now, finding a common denominator, we get

$$F(s) = \frac{a}{s^2}\cdot\frac{s^2+4}{s^2+4} - \frac{1/4}{s^2+4}\cdot\frac{s^2}{s^2}$$

$$= \frac{a(s^2+4) - 1/4\, s^2}{s^2(s^2+4)} = \frac{(a-1/4)s^2 + a\cdot 4}{s^2(s^2+4)}$$

$a = 1/4$

$b = 2$

$c = 4$

Final answer: If $a = 1/4$, $b = 2$, and $c = 4$, then $f(t) = \mathcal{L}^{-1}\{F(s)\}$ and $F(s) = \mathcal{L}\{f(t)\}$.