## Math 2340 Warm-Up Problems

1. Is the function $\mu(x)=e^{x}$ an integrating function for the differential equation

$$
x y^{\prime}+y=x^{4}, \quad x>0 ?
$$

2. Find the general solution to the differential equation

$$
x y^{\prime}-y=x^{3} e^{-x}
$$

Hint: make sure that the differential equation is linear and in standard linear form.
3. Last time, we discussed how to solve linear, first-order, differential equations using an linear integrating function. But there is another way that we can solve this problem: variation of parameters. As an example, consider the following ODE:

$$
\begin{equation*}
\frac{d r}{d t}+2 t r=g(t) \tag{1}
\end{equation*}
$$

(a) Under what conditions is $r(t)=0$ a solution to the ODE?
(b) If $g(t)=0$, solve the differential equation given by (1). Let's call this solution $r_{h}(t)$.
(c) Now, let's solve (1) with $g(t)=2$. To do this, assume that the constant of integration in the previous part is now an unknown function of $t$. In other words, let you $c \rightarrow v(t)$, where $v(t)$ is unknown. Can you use this information to solve for $r(t)$ when $g(t)=2$ ?

Hint: you are guessing a form for the solution that should look like $r(t)=v(t) e^{-t^{2}}$ where $v(t)$ is unknown. If $r(t)$ is a solution, what should you do?

