12. (15 points) Find the solution to the initial value problem given by

$$\vec{\mathbf{u}}' = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix} \vec{\mathbf{u}} + \begin{bmatrix} e^{4t} \\ e^{2t} \end{bmatrix}, \quad \vec{\mathbf{u}}(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Pick a method, any method!

I'm going w/ Laplace Transforms!

Rule # 2

$$s\,U(s) - \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 4 \end{bmatrix} U(s) + \begin{bmatrix} \frac{1}{s-4} \\ \frac{1}{s-2} \end{bmatrix}$$

Solve for $U(s)$

$$\begin{bmatrix} s-2 & 1 \\ -1 & s-4 \end{bmatrix} U(s) = \begin{bmatrix} \frac{1}{s-4} \\ \frac{1}{s-2} \end{bmatrix} \rightarrow U(s) = \frac{1}{s^2-6s+9} \begin{bmatrix} s-4 & -1 \\ 1 & s-2 \end{bmatrix} \begin{bmatrix} \frac{1}{s-4} \\ \frac{1}{s-2} \end{bmatrix}$$

$$\rightarrow U(s) = \frac{1}{(s-3)^2} \cdot \begin{bmatrix} 1 - \frac{1}{s-2} \\ \frac{1}{s-4} + 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{(s-3)^2} - \frac{1}{s-2} \cdot \frac{1}{(s-3)^2} \\ \frac{1}{(s-4)} \cdot \frac{1}{(s-3)^2} + \frac{1}{(s-3)^2} \end{bmatrix}$$

use convolutions or partial fractions.

Find Inverse L.T.

$$\vec{u}(t) = \begin{bmatrix} te^{3t} - e^{2t} * te^{3t} \\ e^{4t} * te^{3t} + te^{3t} \end{bmatrix}$$

$$\boxed{\vec{u}(t) = \begin{bmatrix} e^{3t} - e^{2t} \\ e^{4t} - e^{3t} \end{bmatrix}}$$

note:

$$\frac{1}{(s-3)^2} - \frac{1}{(s-2)(s-3)^2} = \frac{s-2-1}{(s-2)(s-3)^2} = \frac{1}{(s-2)(s-3)}$$

easier to deal with.

Points earned: ______ out of a possible 15 points