Solve $\vec{u}' = A\vec{u}$ where

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 1 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 3-\lambda & 1 \\ -1 & 1-\lambda \end{bmatrix}$$

$\boxed{\text{E-Vals}}$

$$\det(A - \lambda I) = (3-\lambda)(1-\lambda) + 1 = 0$$

$$\Rightarrow 4 - 4\lambda + \lambda^2 = 0$$

$\lambda = 2$ <u>repeated.</u>

$\boxed{\text{E-Vects}}$

$\boxed{\text{1st fund sol}}$ $\lambda = 2$

$$\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad v_1 + v_2 = 0$$

$$\vec{v}^{(1)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad \vec{u}^{(1)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{2t}$$

$\boxed{\text{2nd fund sol}}$

$$(A - \lambda I)\vec{w} = \vec{v}^{(1)}$$

$$\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow \begin{matrix} w_1 + w_2 = 1 \\ -w_1 - w_2 = -1 \end{matrix}$$

If w_1 any const, then
$w_2 = 1 - w_1$

$$\vec{w} = \begin{bmatrix} k \\ 1-k \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + k \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\vec{u}^{(2)} = \vec{v}^{(1)} t e^{2t} + \vec{w} e^{2t}$$

$$= \begin{bmatrix} 1 \\ -1 \end{bmatrix} t e^{2t} + \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{2t} + k \underbrace{\begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{2t}} \right)$$

↳ 1st fund-sol, so we could just choose $k=0$.

$\boxed{\text{Final ans}}$

<u>General Solution</u>

$$\longrightarrow \vec{u}(t) = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{2t} + c_2 \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} t e^{2t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{2t} \right)$$

or

$$\longrightarrow \vec{u}(t) = \left[\begin{array}{c:c} e^{2t} & t e^{2t} \\ \hdashline -e^{2t} & -t e^{2t} + e^{2t} \end{array} \right] \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$