Choosing a method to solve various ODEs

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Choose the Solution Method

$$y' + y = 0$$

Order? First-Order

Linear/Nonlinear? Linear

Is it separable? YES!

Solution Method: Separate & Integrate!

Choose the Solution Method

$$y'' + y = 0$$

Order? Second-Order

Linear/Nonlinear? Linear

Constant Coefficient? Yes!

Homogeneous? Yes!

Solution Method: Guess $y = e^{\lambda x}$ and solve for λ.

Alternative Method: Laplace Transforms, Series, ...

Choose the Solution Method

$$y'' + y = \cos(x)$$

Order? Second-Order

Linear/Nonlinear? Linear

Constant Coefficient? Yes!

Homogeneous? No.

Solution Method: Guess $y = e^{\lambda x}$ and solve for λ to find the homogeneous solution. Then use either the Method of Undetermined Coefficients or VOP to find the particular solution.

Alternative Method: Laplace Transforms, Series, ...

Choose the Solution Method

$$y' + 2xy = \cos(x)$$

Order: First-Order

Linear/Nonlinear: Linear

Is it separable? Nope

Solution Method: Linear Integrating Factor

Alternative Method: Laplace Transforms, Make Exact, Series, ...

Choose the Solution Method

$$(x-2)y'' + xy' + y = 0, \qquad \text{where } y(0) = 1,\ y'(0) = 2.$$

Order? Second-Order

Linear/Nonlinear? Linear

Constant Coefficient? No

Do I know one fundamental solution? No

Regular point or Regular Singular Point? Regular Point.

Solution Method: Use the fact that $y(x)$ has a Taylor Series of the form $y = \sum_{n=0}^{\infty} a_n (x-0)^n$ and solve for all of the a_n's.

Choose the Solution Method

$$f'(t) = 3f(t) + 2g(t)$$

$$g'(t) = 2f(t) - g(t)$$

Order: First-Order System

Linear/Nonlinear: Linear

Is it homogeneous? Yes.

Is it constant coefficient? Yes

Solution Method: Write system as $\vec{\mathbf{u}}' = \mathbf{A}\vec{\mathbf{u}}$ and let $\vec{\mathbf{u}} = \vec{\mathbf{v}}e^{\lambda t}$. Solve for the eigenvalues and eigenvectors.

Choose the Solution Method

$$\left(\sin(x) + x^2 e^y - 1\right) y' + \left(y\cos(x) + 2xe^y\right) = 0$$

$$\underbrace{\left(y\cos(x) + 2xe^y\right)}_{f_x = M} + \underbrace{\left(\sin(x) + x^2 e^y - 1\right)}_{f_y = N} y' = 0$$

Order: First-Order

Linear/Nonlinear: Nonlinear

Is it separable? No.

Is it exact? Does $f_{xy} = f_{yx}$ (or $M_y = N_x$)? Yes

Solution Method: Exact! The implicit solution $f(x, y) = c$ is given by solving $f_x = M$ and $f_y = N$ (system of PDEs)

Choose the Solution Method

$$y''' + yy' - cy' = 0$$

THIS IS A NONLINEAR 3rd order ODE!!!

You can take Math 391 next quarter (Asymptotics) to learn all about it!

This is a really good time to take it. Next time it will be offered is in 2015, and it's the perfect follow-up to this class.

Choose the Solution Method

$$\left(\sin(x) + x^2 e^y - 1\right) y' + \left(y\cos(x) + 2x^3 e^y\right) = 0$$

$$\underbrace{\left(y\cos(x) + 2x^3 e^y\right)}_{f_x = M} + \underbrace{\left(\sin(x) + x^2 e^y - 1\right)}_{f_y = N} y' = 0$$

Order: First-Order

Linear/Nonlinear: Nonlinear

Is it separable? No.

Is it exact? Does $f_{xy} = f_{yx}$ (or $M_y = N_x$)? No

Try to make it exact. Can you find a function μ to multiply the ODE by to make it exact?I give up!

Solution Method: Try direction fields to get an idea. If you have initial conditions, numerically solve!

Choose the Solution Method

$$u'(t) = (t-1)u(t) + 2v(t), \qquad u(0) = 1$$

$$v'(t) = 2u(t) + t^2 v(t), \qquad v(0) = 1$$

Order: First-Order System

Linear/Nonlinear: Linear

Is it homogeneous? Yes.

Is it constant coefficient? No.

Solution Method: Perhaps consider a series solution method where

$$u(t) = \sum_{n=0}^{\infty} a_n t^n, \qquad v(t) = \sum_{n=0}^{\infty} b_n t^n.$$

This should give two equations for each of the "two" unknowns.