MATH 234 - EXAM #2

SPRING QUARTER 2012

Name: ____

Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported to the student misconduct board. By signing the space below, you agree that you will not give nor receive any unauthorized aid during the exam.

Signature: ____

Question:	1	2	3	4	Total
Points:	20	25	40	15	100
Score:					

Question:	5	Total
Bonus Points:	10	10
Score:		

Final Score: ________ out of 100 points = ________%

- This exam has a total of 5 problems.
- Please complete the exam on your own paper (or on the extra paper provided). Please staple this page as the cover of the exam, and return the exam questions to the proctor.
- **You are not allowed to use a calculator on this exam**.
- You should show/explain your work on all problems to receive full credit. The correct answer with no supporting work *will* result in no credit.
- If you have a question, please come up to the front.
- There is extra paper available up front for scratch work, or if you run out of space.
- **Clearly indicate your final answer by placing a box around it.**
- You will have 85 minutes to complete your exam.
- There is a table of Laplace Transformations at the end of the exam. If you use any item on the table of transforms, please refer to the rule # in your exam.

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ANSWER THE FOLLOWING QUESTIONS

1. Answer the following questions regarding the Laplace Transform.
   (a) (10 points) Consider the function
   $$g(t) = 3u_1(t) - u_2(t)\,(5-t)\,.$$
   Sketch the graph of $g(t)$ for $0 \leq t \leq 5$ and determine the Laplace transform of $g(t)$ via the definition. **You must show your work to receive full credit.**
   (b) (10 points) Find $f(t)$ if
   $$\mathcal{L}\{f(t)\} = \frac{1}{s^3 + 2s^2 + s}.$$

2. (25 points) Solve the following initial value problem:
   $$y'' - 4y' + 13y = e^t * \delta(t-4), \qquad \text{where} \quad y(0) = 6, \quad y'(0) = 12.$$
   Note: $*$ represents the convolution and not multiplication.

3. Consider the following differential equation
   $$(3-x)y'' + y' + (x^2 - 1)y = 0$$
   (a) (10 points) Determine all of the regular points and singular points of the differential equation. If there are any singular points, determine if they are regular singular points or not.
   (b) (25 points) Solve the above differential equation by means of a power series centered at $x = 1$. Find the recurrence relation and the first two terms in *each* of the two fundamental solutions.
   (c) (5 points) What can be said about the radius of convergence of the solution obtained in part (b)?

TRUE/FALSE

4. Answer the following **TRUE** or **FALSE**. If you answer **true**, you must clearly justify your answer. If you answer **false**, you must provide the correct version on the statement given and fully justify your new statement.
   (a) (5 points) **True/False** The convolution of $f(t)$ and $g(t)$ satisfy the following property:
   $$f(t) * g(t) = -g(t) * f(t)$$

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(b) (5 points) **True/False** Consider the piecewise function $f(t)$ given by

$$f(t) = \begin{cases} 0 & 0 \leq t < \frac{\pi}{2} \\ \cos(t) & \frac{\pi}{2} \leq t < \frac{3\pi}{2} \\ 0 & t \geq \frac{3\pi}{2} \end{cases}$$

The function $f(t)$ can be written using unit-step notation as

$$f(t) = \cos(t) u_{\frac{\pi}{2}}(t).$$

(c) (5 points) **True/False** Consider the differential equation

$$\left(4 + x^2\right) y'' - xy = 0.$$

The lower bound on the radius of convergence for the series solution centered at $x = 1$ is given by $\rho = \sqrt{5}$.

Bonus

5. (10 points (bonus)) **Bonus Problem!** Determine the recurrence relationship for the series solution to the following differential equation centered at $x = 0$.

$$y'' - xy = \ln|1 - x|$$

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}, \qquad s > 0$
2. e^{at}	$\frac{1}{s-a}, \qquad s > a$
3. $t^n, \quad n = \text{positive integer}$	$\frac{n!}{s^{n+1}}, \qquad s > 0$
4. $t^p, \qquad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \qquad s > 0$
5. $\sin at$	$\frac{a}{s^2+a^2}, \qquad s > 0$
6. $\cos at$	$\frac{s}{s^2+a^2}, \qquad s > 0$
7. $\sinh at$	$\frac{a}{s^2-a^2}, \qquad s > \|a\|$
8. $\cosh at$	$\frac{s}{s^2-a^2}, \qquad s > \|a\|$
9. $e^{at} \sin bt$	$\frac{b}{(s-a)^2+b^2}, \qquad s > a$
10. $e^{at} \cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \qquad s > a$
11. $t^n e^{at}, \quad n = \text{positive integer}$	$\frac{n!}{(s-a)^{n+1}}, \qquad s > a$
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \qquad s > 0$
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$
14. $e^{ct}f(t)$	$F(s-c)$
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \qquad c > 0$
16. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$
19. $(-t)^n f(t)$	$F^{(n)}(s)$