MATH 234 - EXAM #2

SPRING QUARTER 2013

Name: ____

Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported. By signing the space below, you agree that you will not give nor receive any unauthorized aid during the exam.

Signature: ____

Question:	1	2	3	4	Total
Points:	10	10	40	40	100
Score:					

Question:	5	Total
Bonus Points:	10	10
Score:		

Final Score: ________ out of 100 points = ________%

- Due to the nature of the problems on this exam, you are asked to solve all problems on your own paper. Please clearly number each problem, and box your final answer.
- You exam consists of this cover sheet, a page of questions, and a table of Laplace transformations. You must turn all of these items in (along with your solutions) to receive credit for your exam.
- **You are not allowed to use a calculator on this exam**.
- You should show/explain your work on all problems to receive full credit. The correct answer with no supporting work *may* result in no credit.
- You must staple all scratch paper to your exam. **Please attach this cover sheet to the top of your exam**.
- There is extra paper available up front for scratch work, or if you run out of space.
- **Clearly indicate your final answer by placing a box around it.**
- You will have 85 minutes to complete your exam.
- Solutions will be posted online at 2:00pm.

GOOD LUCK!!!

1. (10 points) Answer the following questions **True / False** . If you answer **TRUE**, you must justify your answers. If you answer **FALSE**, provide the correct statement.

   (a) **True / False.** If $Y(s) = \dfrac{1}{s^4 + s^2}$, then
   $$y(t) = \mathcal{L}^{-1}\{Y(s)\} = t\sin(t).$$

   (b) **True / False.** The point $x_0 = 1$ is a **regular singular point** (RSP) of the differential equation
   $$(x-1)^2 y'' + (x+1)y' + y = 0.$$

2. (10 points) **Short Answer / Short Response.**

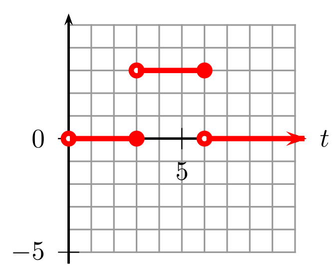
   (a) Using the definition, determine the Laplace transform of $g(t) = u_1(t) \cdot \left(t^2 + 1\right)$.

   (b) If possible, give an example of a differential equation with a Taylor series solution $y(x)$ (centered at $x_0 = 1$) that will converge for x that satisfy $|x - 1| < 2$. If it is not possible, explain why. Use complete sentences.

3. (40 points) Solve the following initial value problems

   (a) $y'' + 7y' + 12y = \sin\left(\dfrac{\pi t}{2}\right) \cdot \delta(t-1)$ where $y(0) = 0$ and $y'(0) = 0$.

   (b) $y'' + 9y = g(t)$ where $y(0) = 1$ and $y'(0) = -1$. The function $g(t)$ is given in the figure below:



4. Consider the following differential equation
   $$xy'' + y' + y = 0.$$

   (a) (5 points) Determine all of the ordinary and singular points of the differential equation. If there are any singular points, determine if they are regular singular points or not.

   (b) (25 points) Solve the above differential equation by means of a power series centered at $x = 1$. Find the recurrence relation and the first two terms in *each* of the two fundamental solutions.

   (c) (5 points) Using your answer from part (b), determine the solution to the corresponding initial value problem $y(1) = 1$, and $y'(1) = 0$.

   (d) (5 points) What can be said about the radius of convergence of the solution obtained in part (c)?

5. (10 points (bonus)) An **integro-differential equation** is an equation where both derivatives and integrals of the unknown function $y(t)$ appear. Find the solution to the following integro-differential equation:
   $$y'(t) + \int_0^t (t-\tau)y(\tau)\,d\tau, \qquad y(0) = 0$$

LAPLACE TRANSFORMATIONS

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}, \quad s > 0$
2. e^{at}	$\frac{1}{s-a}, \quad s > a$
3. $t^n, \quad n =$ positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$
4. $t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5. $\sin at$	$\frac{a}{s^2 + a^2}, \quad s > 0$
6. $\cos at$	$\frac{s}{s^2 + a^2}, \quad s > 0$
7. $\sinh at$	$\frac{a}{s^2 - a^2}, \quad s > \lvert a \rvert$
8. $\cosh at$	$\frac{s}{s^2 - a^2}, \quad s > \lvert a \rvert$
9. $e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}, \quad s > a$
10. $e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}, \quad s > a$
11. $t^n e^{at}, \quad n =$ positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13. $u_c(t) f(t-c)$	$e^{-cs} F(s)$
14. $e^{ct} f(t)$	$F(s-c)$
15. $f(ct)$	$\frac{1}{c} F\left(\frac{s}{c}\right), \quad c > 0$
16. $\int_0^t f(t-\tau) g(\tau)\, d\tau$	$F(s) G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $f^{(n)}(t)$	$s^n F(s) - s^{n-1} f(0) - \cdots - f^{(n-1)}(0)$
19. $(-t)^n f(t)$	$F^{(n)}(s)$