MATH 234 - EXAM #1

SPRING QUARTER 2013

Name: ______________________________

Any student found engaging in academic misconduct will receive a score of 0 on this exam. All suspicious behavior will be reported. By signing the space below, you agree that you will not give nor receive any unauthorized aid during the exam.

Signature: ______________________________

Page:	3	4	6	7	8	9	10	11	Total
Points:	8	12	10	10	15	15	15	15	100
Score:									

Page:	12	Total
Bonus Points:	10	10
Score:		

Final Score: ________ out of 100 points = ________%

- Check that your exam has a total of 14 pages and 10 problems.
- **You are not allowed to use a calculator on this exam**.
- You should show/explain your work on all problems to receive full credit. The correct answer with no supporting work *may* result in no credit.
- If you have a question, please come up to the front.
- There is extra paper available up front for scratch work, or if you run out of space.
- **Clearly indicate your final answer by placing a box around it.**
- You will have 85 minutes to complete your exam.

GOOD LUCK!!!

TRUE / FALSE

For the **True / False** questions **you must justify your answers.**

1. (4 points) **True / False.** It is possible to find an integrating function $\mu(y)$ (μ depends only on y) such that the following differential equation

$$x^2 + 2x + y^2 + 2yy' = 0$$

becomes exact when multiplied by μ.

2. (4 points) **True / False.** There exists a unique solution to the following initial value problem

$$xy' = y, \qquad y(0) = 0.$$

Points earned: ____________ out of a possible 8 points

MATCHING/MULTIPLE CHOICE

3. Consider the plots of solutions to initial value problems for various second-order differential equations of the form
$$ay'' + by' + cy = g(x),$$
(plots given on the next page).

If possible, match the following differential equations with a plot that best matches a solution to a corresponding initial value problem. If it is not possible to match the differential equation with a corresponding figure, use one of the blank grids on the next page to sketch a suitable graph corresponding to a non-trivial (non-zero) solution to an initial value problem.

(a) (4 points) $y'' - 2y' + 2y = 0$

A solution corresponds to Figure ____________

(b) (4 points) $3y'' + 17y' + 10y = 0$

A solution corresponds to Figure ____________

(c) (4 points) $y'' + y = \sin(2t)$

A solution corresponds to Figure ____________

Points earned: ______________ out of a possible 12 points

Figure A

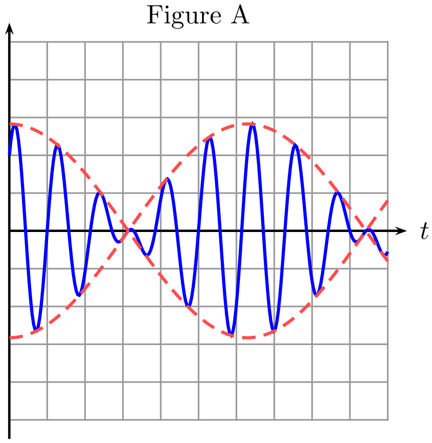


Figure B

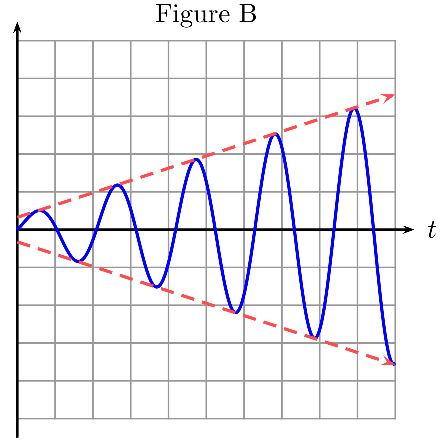


Figure C

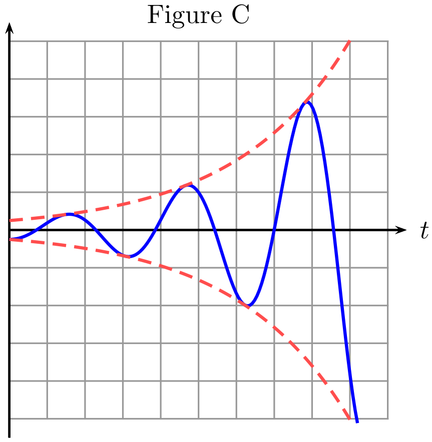


Figure D

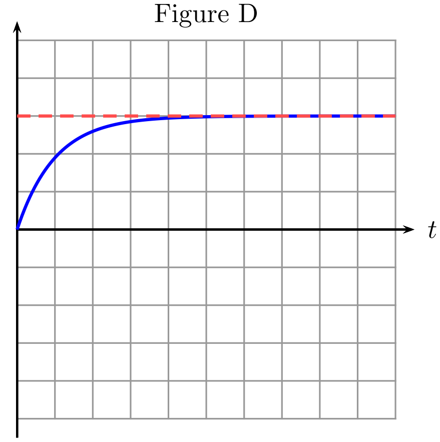


Figure E

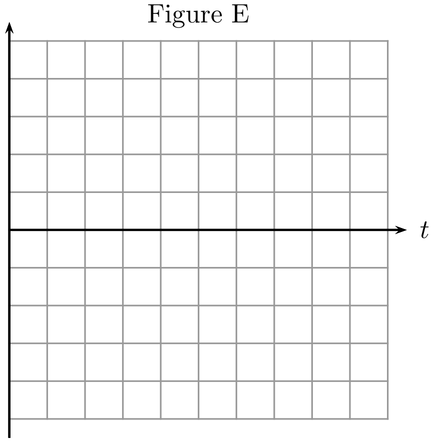
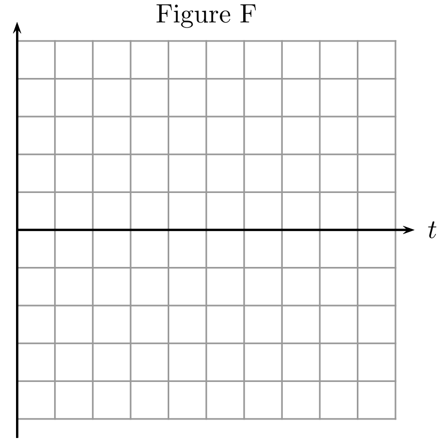


Figure F



Points earned: ____________ out of a possible 0 points

FIND THE SOLUTION

Find the general solution to each of the following differential equations.

You may leave your solution in implicit form if you like.

4. (10 points) $4y'' - 8y' - 4y = 0$

Points earned: ______________ out of a possible 10 points

5. (10 points) $r'(t) = \dfrac{t - e^{-t}}{r(t) + e^{r(t)}}$

Points earned: ______________ out of a possible 10 points

6. (15 points) $y'' + 2y' + y = \dfrac{e^{-t}}{t}$

> In order to solve this ODE, you would need to use variation of parameters for a second order ODE. This topic will not be included on the Winter 2015 Exam #1.

Points earned: ____________ out of a possible 15 points

Solve each of the following initial value problems.

You may leave your solution in implicit form if you like.

7. (15 points) Solve the initial value problem $xy' = y + x^2e^{-x}, \quad y(1) = 4.$

Points earned: ____________ out of a possible 15 points

8. (15 points) $\dfrac{dy}{dx} = \dfrac{3x + xy^2}{y + x^2y}, \qquad y(1) = 3$

Points earned: ____________ out of a possible 15 points

9. (15 points) $q'' + 4q' + 13q = e^{-2t}, \quad q(0) = 1,\ q'(0) = 2$

Points earned: ______________ out of a possible 15 points

BONUS

10. (10 points (bonus)) Let both P and Q be functions that are continuously differential in both x and y (i.e. $Q(x, y)$ and $P(x, y)$ both have continuous derivatives with respect to x and y). If $P_x = Q_y$ and $Q_x = -P_y$, show that the differential equation

$$P(x, y) + Q(x, y)\frac{dy}{dx} = 0$$

is not exact in general, but becomes exact when multiplied by

$$\frac{1}{P^2 + Q^2}.$$

Bonus points earned: __________ out of a possible 10 points

TABLE OF INTEGRALS

(1) $\displaystyle\int \sin(x)\ dx = -\cos(x) + c$

(2) $\displaystyle\int \cos(x)\ dx = \sin(x) + c$

(3) $\displaystyle\int \sin^2(x)\ dx = \frac{1}{2}\left(x - \sin(x)\cos(x)\right) + c$

(4) $\displaystyle\int \cos^2(x)\ dx = \frac{1}{2}\left(x + \sin(x)\cos(x)\right) + c$

(5) $\displaystyle\int \sin^2(x)\cos(x)\ dx = \frac{1}{3}\sin^3(x) + c$

(6) $\displaystyle\int \cos^2(x)\sin(x)\ dx = -\frac{1}{3}\cos^3(x) + c$

(7) $\displaystyle\int \frac{1}{\sin^2(x)}\ dx = -\frac{\cos(x)}{\sin(x)} + c$

(8) $\displaystyle\int \frac{1}{\cos^2(x)}\ dx = \frac{\sin(x)}{\cos(x)} + c$

(9) $\displaystyle\int \frac{ax}{1+bx^2}\ dx = \frac{a}{2b}\ln(1+bx^2) + c$

(10) $\displaystyle\int xe^x dx = (x-1)e^x + c$

(11) $\displaystyle\int x^2e^x\ dx = \left(x^2 - 2x + 2\right)e^x + c$

(12) $\displaystyle\int e^{ax}\cos(bx)\ dx = \frac{e^{ax}}{a^2+b^2}\left(a\cos(bx) + b\sin(bx)\right)$

(13) $\displaystyle\int e^{ax}\sin(bx)\ dx = \frac{e^{ax}}{a^2+b^2}\left(a\sin(bx) - b\cos(bx)\right)$