Project Life Cycle Stages
### Recall Pert/CPM Estimates

<table>
<thead>
<tr>
<th>Task</th>
<th>Immediate Predecessors</th>
<th>Optimistic</th>
<th>Most Likely</th>
<th>Pessimistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>None</td>
<td>3</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>B</td>
<td>None</td>
<td>2</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>C</td>
<td>A</td>
<td>6</td>
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</tr>
<tr>
<td>D</td>
<td>A</td>
<td>2</td>
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<td>E</td>
<td>C</td>
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<td>17</td>
</tr>
<tr>
<td>F</td>
<td>D</td>
<td>3</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>G</td>
<td>B</td>
<td>3</td>
<td>9</td>
<td>27</td>
</tr>
<tr>
<td>H</td>
<td>E,F</td>
<td>1</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>I</td>
<td>G,H</td>
<td>4</td>
<td>19</td>
<td>28</td>
</tr>
</tbody>
</table>
More PERT/CPM Estimates

- **Activity Time Estimates for beta distribution**
  - a: optimistic time, usually about 1 percent chance activity can be completed within \( a \)
  - b: most likely time
  - c: pessimistic time, usually about 1 percent chance that activity will take longer than \( c \).

- Activity j’s expected time (b distribution)

\[
ET(j) = \frac{a + 4b + c}{6}
\]

- Activity j’s variance (b distribution)

\[
\sigma_j^2 = \left(\frac{c - a}{6}\right)^2
\]
PERT Notation

\[ D \quad \text{the desired project duration} \]

\[ T_e \quad \text{expected project duration} \]

\[ Te = \sum_{j \in \text{any one critical path}} ET(j) \]

Critical path variance =

\[ \sum_{j \in \text{any one critical path}} \sigma_j^2 \]

\[ Z = \text{number of standard normal deviates from mean} = \frac{D - T_e}{\sqrt{\text{Critical Path Variance}}} \]
What is the probability of finishing this project in less than 53 days?

\[ Z = \frac{D - T_E}{\sqrt{\sum \sigma_{cp}^2}} \]
PERT Network

Duration = 54 Days

\[ \sigma^2(A) = 2^2 = 4 \]

\[ \sigma^2(C) = 16 \]

\[ \sigma^2(E) = 4 \]

\[ \sigma^2(H) = 1 \]

\[ \sigma^2(I) = 16 \]
\[ \sigma_j^2 = \left( \frac{\text{Pessim}(j) - \text{Optim}(j)}{6} \right)^2 \]

<table>
<thead>
<tr>
<th>Task</th>
<th>Optimistic</th>
<th>Most Likely</th>
<th>Pessimistic</th>
<th>Variance</th>
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<tr>
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<td>12</td>
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<td>16</td>
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<td>E</td>
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<td>11</td>
<td>17</td>
<td>4</td>
</tr>
<tr>
<td>F</td>
<td>3</td>
<td>6</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>3</td>
<td>9</td>
<td>27</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>1</td>
<td>4</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>I</td>
<td>4</td>
<td>19</td>
<td>28</td>
<td>16</td>
</tr>
</tbody>
</table>

(Sum the variance along the critical path.) \( = 41 \)
There is a 43.6% probability that this project will be completed in less than 53 weeks.
Crashing Activities

- Activity & project duration may be influenced by resource allocation decisions…more resources—less time needed
- Sometimes, we can expedite completion of a task by assigning more people or equipment. Thus, total direct costs tend to vary inversely with project duration. Indirect costs, however, tend to increase with overall project completion time.
Time-Cost Tradeoffs in CPM

- **Basic Assumption:** Some activities can be expedited, at a cost
- **Why accelerate an activity?**
  - Complete the task in a shorter duration, thereby reducing the duration of the critical path, thereby reducing the length of the project
    - Avoid late penalties
    - Earn early completion incentive payments
- **Time Cost Problem:** Determine the optimum project duration based on time-cost tradeoffs
Project & Activity Costs

• **PROJECT DIRECT COSTS**: Directly assigned to work package or activity
  – “Normal” costs for “normal” time
    • Direct labor expenses
    • Materials
    • Equipment
    • Subcontractors

• **INDIRECT EXPENSES**: Cannot be associated with any work package or activity
  – Overhead expenses
    • including supervisory expenses
    • administration
    • *contractual penalties or early completion incentives*
  – Consulting
Time & Costs: Normal vs. Crash

• For the time-only CPM project schedule, we typically assume that activity duration is fixed at its NORMAL TIME, or the duration with the lowest direct activity cost (i.e., NORMAL COST).

• However, some activities may be expedited if higher resource levels are available. The shortest activity duration is called CRASH TIME. The cost to complete an activity in that amount of time is called CRASH COST.
In theory, the normal or expected duration of a task can be reduced by assigning additional resources to the task.

Slope \(b_j\) = Increase in cost by reducing task by one time unit.
Selecting Activities to Crash

• Cost Slope = Rise / Run
• Cost Slope = CC – NC / NT – CT
• \[ \frac{CC - NC}{NT - CT} \]
• where,
  – CC = Crash Cost
  – NC = Normal Cost
  – NT = Normal Time
  – CT = Crash Time

• Calculate for each activity on Critical Path

\[ \frac{CC - NC}{NT - CT} = \frac{800 - 400}{10 - 5} \]
\[ = \frac{400}{5} = 80 \text{ per unit of time} \]
Balancing Overhead & Direct Costs

Project Duration

Crash Time  Minimum Cost Solution  Normal Time

Cost

Total Cost

Indirect (overhead) Costs

Direct Costs
Software Project Schedules

“Observe that for the programmer, as for the chef, the urgency of the patron may govern the scheduled completion of the task, but it cannot govern the actual completion. An omelet, promised in ten minutes, may appear to be progressing nicely. But when it has not set in ten minutes, the customer has two choices--wait or eat it raw. Software customers have the same choices.

The cook has another choice; he can turn up the heat. The result is often an omelet nothing can save--burned in one part, raw in another.”

Time/Cost Trade-off Analysis

• You might think that total project costs will increase when we begin to crash activities.
• But, total project costs consist of both indirect (project-based) costs (PBC) and direct (activity-based) costs (ABC).
  – ABC (direct) go up when we crash activities in an effort to finish the project early.
  – But, PBC (the indirect costs) go down if we finish the project early.
Which Activities are the Best Candidates for Crashing?

- Any activity that is on the critical path
- Activities with relatively long durations
- Bottleneck activities (that appear on multiple critical paths)
- Activities with relatively low costs to crash
- Activities that are not likely to cause quality problems if crashed
- Activities that occur relatively early in the schedule and are labor intensive
Options for Crashing Activities

- Adding Resources
- Outsourcing Project Work
- Overtime
- Establishing Core Project Team
- Temporary Fixes
- Fast-Tracking
- Critical Chain PM
- Brainstorming
- Reducing Scope
- Phasing Project Deliverables
Potential Problems with Crashing

- Reduced flexibility and less margin for error → increased risk of failure to complete project on time
- Raises potential for poor quality
- Increases potential for staff burnout, stress, and turnover (from what Yourdon calls Death March projects)
- Raises activity-based costs
- May negatively affect other projects
- May create unrealistic expectations for future projects
- Hard to know true indirect costs
Time-Cost Tradeoff Heuristic

1. Set each activity duration to its normal time.
2. Determine the critical path(s) and project duration based on the current activity times.
3. Calculate total direct costs and indirect costs for the current schedule.
4. Choose an activity or activities that can be expedited to reduce project duration by one time unit. Use cost slopes and critical path information to guide your choice. If no further reduction in project duration is possible, go to step 5. Otherwise, go to step 2.
5. Plot the project's direct and indirect costs for each possible duration. Select the duration that minimizes total costs.
## Network Crashing Example

<table>
<thead>
<tr>
<th>ACTIVITY ID</th>
<th>Normal Time</th>
<th>Normal COST</th>
<th>Max Allow Crash Time</th>
<th>Crash COSTS</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>$50</td>
<td>1</td>
<td>$20</td>
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<tr>
<td>C</td>
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<td>60</td>
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<td>30</td>
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<tr>
<td>D</td>
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<td>G</td>
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<td>70</td>
<td>0</td>
<td>00</td>
</tr>
</tbody>
</table>
Network Crashing Example
Network Crashing Example
Network Crashing Example

Total Direct Cost = $495
Total Duration = 23 units
Activities Changed
D
$25 __  ___
Network Crashing Example

Total Direct Cost = $525
Total Duration = 22 units
Activities Changed

$30
Network Crashing Example

A (2x) → B (6) → E (7) = 
D (9) → C (9x) → F (4x) → G (6) = 

___  ____  ____
## Network Crashing Example

<table>
<thead>
<tr>
<th>PROJECT DURATION</th>
<th>DIRECT COSTS</th>
<th>INDIRECT COSTS</th>
<th>TOTAL COSTS</th>
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<tbody>
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<td>200</td>
<td>810</td>
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</table>
Crashing Summary

• In planning, crashing can pull your project into the delivery window
• Once started, a late project can get pulled back in to schedule compliance
• Crashing can increase the size of the “critical core,” thereby reducing flexibility
• Capability in MS Project