

Final Exam Practice

Math 120 (Doherty)

Name _____

1. Solve each equation for x and state the solution set. Also state whether the equation is inconsistent, conditional, or an identity.

(a) $x(x - 6) = 27$

(e) $c = \frac{a}{b - x}$

(b) $x^2 + 10 = 6x$

(f) $\sqrt{5x + 14} = x$

(c) $\frac{1}{2x + 5} + \frac{1}{x - 1} = \frac{1}{2x^2 + 3x - 5}$

(g) $\sqrt{x + 1} = \sqrt{x} = 2$

(d) $|7 - 2x| = 19$

(h) $(x + 1)^{-2/3} = 2$

2. Sketch a graph of each of the following. State whether y is a function of x .

(a) $2y + x = 4$

(b) $(x + 1)^2 + (y - 2)^2 = 9$

3. Find the center and radius of the circle described by the equation $x^2 + y^2 - 4x + 2y + 1 = 0$.

4. Given the points $A(-3, 4)$ and $B(1, 3)$, find the following:

(a) the distance between A and B ;

(b) the midpoint of the line segment joining A and B ;

(c) the equation of the circle with center A , and passing through the point B ;

(d) the equation of the circle having a diameter with endpoints at A and B ;

(e) the equation of the line containing A and B .

5. Given $f(x) = 3x^2 - 2x$, find and simplify:

(a) $f(-3)$

(c) $f(x - 2)$

(d) $\frac{f(x + h) - f(x)}{h}$

(b) $f(4x)$

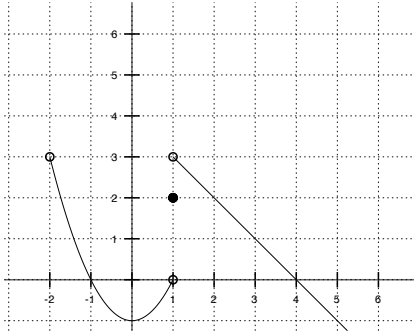
6. State the domain of the following functions in interval notation:

(a) $f(x) = \sqrt{2x - 3}$

(c) $h(x) = \sqrt{\frac{x + 2}{x - 3}}$

(b) $g(x) = \frac{5x}{x^2 + x - 12}$

7. Use the graph of $f(x)$, given below, to answer the following:



(a) Find $f(-1)$ and $f(3)$.

(b) Find the x -intercept(s) of $f(x)$.

(c) Find the y -intercept(s) of $f(x)$.

(d) For what values of x is $f(x) > 0$?

(e) State the domain and range of $f(x)$ in interval notation.

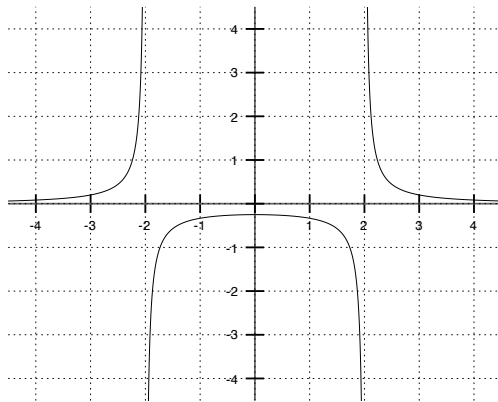
(f) Find a formula for $f(x)$ (hint: it's a piecewise function).

8. Are the following functions even, odd, or neither? Explain your answer.

(a) $f(x) = x^2 + 3x - 1$

(b) $g(x) = 5x^3 - x$

(c) The function graphed below:



9. Sketch a graph of each of the following. Which one doesn't have an inverse function? Why?

(a) $f(x) = 2\sqrt[3]{x}$

(b) $g(x) = -(x - 3)^2$

(c) $h(x) = -\sqrt{x} + 1$

10. Find the formula of the function obtained after applying the following three transformations to $y = |x|$:

- reflect the graph about the x -axis;
 - shrink by a factor of $\frac{1}{3}$;
 - shift to the left 3 units.
11. Let $f(x) = 2(x - 1)^2 - 4$
- (a) Find the vertex of f .
 - (b) Find all the intercepts of f .
 - (c) Sketch the graph of f .
12. Find the vertex of $f(x) = -\frac{1}{2}x^2 - 3x + 1$. Is the vertex a minimum of this function, or a maximum?
13. If a toy rocket is fired up into the air from the top of a 10 foot ladder with an initial velocity of 72 ft/s, its height above the earth after t seconds is given by $f(t) = -16t^2 + 72t + 10$.
- (a) What is the height of the rocket after 1 second?
 - (b) After how many seconds does the rocket reach its maximum height?
 - (c) What is the maximum height reached?
 - (d) After how many seconds does the toy rocket hit the ground? Give an exact answer.
14. Let $f(x) = \sqrt{2 - x}$ and $g(x) = 1 + 2x$. Determine each of the following (simplify as much as possible):
- (a) $(f + g)(-1)$
 - (b) $(f - g)(1)$
 - (c) $\frac{f}{g}(x)$, and its domain
 - (d) $(f \circ g)(x)$
 - (e) $(g \circ f)(x)$
15. Sketch a graph of
- $$f(x) = \begin{cases} -|x| + 1 & \text{if } x \leq 2 \\ \sqrt{x - 2} & \text{if } x > 2 \end{cases}$$
16. Let $f(x) = -2(x + 4)(x - 3)^2$.
- (a) Find the zeros of f and their multiplicities.
 - (b) Find the y -intercept of $f(x)$.
 - (c) Describe the end behavior of $f(x)$.
 - (d) Sketch a graph of $f(x)$. Clearly label all intercepts.
17. Sketch a graph of $P(x) = x^4 - 17x^2 + 16$. Clearly label all intercepts.
18. For each function given below, find the intercepts and asymptotes. Then sketch a graph of the function.

$$(a) f(x) = \frac{x}{x^2 - x - 2}$$

$$(b) g(x) = \frac{x^2 - 1}{x^2 + 3x + 2}$$

19. Solve each of the following inequalities. State the solution set in interval notation.

$$(a) x^2 > 3(x + 6)$$

$$(c) \frac{2x - 5}{x + 1} < 0$$

$$(b) x(x^2 - 9) \leq 0$$

$$(d) \frac{5}{x - 3} > \frac{3}{x + 1}$$

20. Use synthetic division to evaluate $P(2)$, if $P(x) = 3x^4 - 2x^2 - 7x + 6$.

21. Let $f(x) = 2x^3 - 9x^2 - 8x + 15$.

(a) List all possible rational zeros of f .

(b) Use synthetic division to show that 1 is a zero of f .

(c) Find all other zeros of f .

22. Let $g(x) = x^4 - 2x^3 + x^2 - 8x - 12$

(a) Given that $x = 2i$ is a zero of g , find all other zeros (real and imaginary) of g .

(b) Factor g completely.

(c) What are the x -intercepts of g ?

23. Let $z = 2 + \sqrt{-9}$ and $w = 4 - i$. Write z in standard form. Then write each of the following in standard form:

$$(a) z + w$$

$$(b) w - z$$

$$(c) z \cdot w$$

24. Find a degree 5 polynomial with real coefficients that has the zeros 2, $5i$, and -3 (the last with multiplicity 2).

25. Let $f(x) = \frac{-2x}{x - 1}$.

(a) Show that f is one-to-one.

(b) Find $f^{-1}(x)$.

(c) Check your answer to (b) by showing that $f(f^{-1}(x)) = x$.

(d) State the range of $f(x)$ by examining the domain of $f^{-1}(x)$.

26. Sketch a graph of the function $f(x) = -2^x + 1$. Then state the domain, range, and horizontal asymptote of f .

27. Sketch a graph of $g(x) = \log_2(x + 3)$. Then state the domain, range, and equation of the vertical asymptote of g .

28. Evaluate each of the following. Give exact answers.

(a) $\log_5(125)$

(c) $10^{\log(\pi)}$

(e) $\log_3(9^{27})$

(b) $\log_7(\frac{1}{49})$

(d) $\log_{1/5}(\frac{1}{125})$

(f) $\ln(e^{\sqrt{13}})$

29. Use your calculator to approximate $\log_5(7)$. Round your answer to three decimal places.

30. Solve for x :

(a) $2^{3x-1} = 16$

(d) $3^{2x+1} = 17$

$\ln(14 - 2x)$

(b) $\frac{e^{x^2}}{(e^4)^x} = e^{12}$

(e) $\log_x(\frac{1}{8}) = 3$

(h) $\log_7(x + 4) + \log_7(x - 2) = 1$

(c) $4e^{3-5x} = 11$

(f) $\log_5(4x + 15) = 3$

(g) $\ln(x + 3) + \ln(x) =$ (i) $3^{1-x} = 7^{-x}$

31. A certain radioactive element decays according to the function $A(t) = A_0e^{-0.045t}$, where t is the time in years and A_0 is the initial amount present.

(a) What is the half-life of this element? (Recall: half-life means the time it takes until half the original material remains.)

(b) If 840 grams are present initially, how much will be left after 100 years?

32. Solve each of the following systems by graphing, and state whether the system is dependent, independent, or inconsistent:

(a)

$$3x - 4y = 0$$

$$y = \frac{3}{4}x + 2$$

(b)

$$x - 2y = -3$$

$$y = \frac{1}{2}x + \frac{3}{2}$$

(c)

$$2x - 3y = -4$$

$$y = -2x + 4$$

33. Solve each of the following systems algebraically:

(a) For this system, also state whether it's independent, dependent, or inconsistent.

$$2x + 3y = 1$$

$$3x - 5y = -8$$

(b)

$$\begin{aligned}x^2 - y^2 &= 9 \\x^2 + y^2 &= 7\end{aligned}$$

(c)

$$\begin{aligned}y &= |x| \\y &= x^2\end{aligned}$$

(d)

$$\begin{aligned}y &= 2x^2 + x - 3 \\6x + y &= 12\end{aligned}$$

34. Determine whether the following sequences are arithmetic, geometric, or neither. If they're arithmetic or geometric, find the formula for the n th term, and find the sum of the first 11 terms.

(a) $0, -1, 4, -9, 16, \dots$ (b) $16, 13, 10, 7, \dots$ (c) $\frac{1}{3}, -\frac{1}{6}, \frac{1}{12}, -\frac{1}{24}, \dots$

35. Write each sum using summation notation:

(a) $\frac{5}{2} - \frac{5}{8} + \frac{5}{32} - \frac{5}{128}$ (b) $1 + 4 + 9 + 25 + \dots + n^2$

36. Find each infinite sum:

(a) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ (b) $\sum_{i=0}^{\infty} (0.98)^i$

37. Find the coefficient of a^3b^4 in the expansion of $(2a - 3b)^7$.