Math 110 Quiz #5 Name ______________________

Instructions: Read carefully. Show your work.

1. (4 pts) State the domain of $\frac{x}{x^2 - x - 2}$, using interval notation.

The only values *not* in the domain are the ones making the denominator zero, i.e., the solutions to $x^2 - x - 2 = 0$ Solve by factoring:

$$(x-2)(x+1) = 0$$

The solutions to this are $x = 2$ and $x = -1$ – these are the disallowed values for x. To write this in interval notation, we simply need to write intervals which cover all real numbers except these ones:

$$(-\infty, -1) \cup (-1, 2) \cup (2\infty)$$

2. (4 pts each) Reduce each rational expression to its lowest terms.

(a) $\frac{a^3bc^3}{a^5b^2c}$

$$\begin{aligned}\frac{a^3bc^3}{a^5b^2c} &= \frac{c^{3-1}}{a^{5-3}b^{2-1}} \\ &= \frac{c^2}{a^2b}\end{aligned}$$

(b) $\frac{5x^2 - 15x + 10}{5x - 10}$

$$\begin{aligned}\frac{5x^2 - 15x + 10}{5x - 10} &= \frac{5(x^2 - 3x + 2)}{5(x-2)} \\ &= \frac{(x-2)(x-1)}{(x-2)} = x - 1.\end{aligned}$$

3. (6 pts each) Perform the indicated operation. State your answer in lowest terms.

(a) $\dfrac{4x-2}{x^2-5x} \div \dfrac{2x^2+9x-5}{x^2-25}$

$$\begin{aligned}\frac{4x-2}{x^2-5x} \div \frac{2x^2+9x-5}{x^2-25} &= \frac{4x-2}{x^2-5x} \times \frac{x^2-25}{2x^2+9x-5} \\ &= \frac{2(2x-1)}{x(x-5)} \cdot \frac{(x-5)(x+5)}{(2x-1)(x+5)} \\ &= \frac{2}{x}\end{aligned}$$

(b) $\dfrac{1}{x} + \dfrac{2}{x-1} - \dfrac{3}{x+2}$

We need to get a common denominator here. Since all three denominators are prime, the LCD is simply $x(x-1)(x+2)$. Now we need to contribute the appropriate parts of the LCD to each fraction:

$$\frac{1}{x} \cdot \frac{(x-1)(x+2)}{(x-1)(x+2)} + \frac{2}{x-1} \cdot \frac{x(x+2)}{x(x+2)} - \frac{3}{x+2} \cdot \frac{x(x-1)}{x(x-1)}$$

This will give all three fractions the same denominator, so we can put everything into one fraction now:

$$\frac{(x-1)(x+2) + 2x(x+2) - 3x(x-1)}{x(x-1)(x+2)}.$$

Simplifying, we get

$$\frac{8x-2}{x(x-1)(x+2)} = \frac{2(4x-1)}{x(x-1)(x+2)}.$$

(c) $\dfrac{5}{x^2+x-2} - \dfrac{x+5}{x^2+2x-3}$

Again, we need a common denominator; we need to factor if we want to find the LCD:

$$\frac{5}{(x+2)(x-1)} - \frac{x+5}{(x+3)(x-1)}.$$

Looking at the denominators, the LCD should be $(x-1)(x+2)(x+3)$. Now we contribute to each fraction:

$$\frac{5}{(x+2)(x-1)} \cdot (x+3)(x+3) - \frac{x+5}{(x+3)(x-1)} \cdot (x+2)(x+2).$$

Put it all into one fraction:

$$\frac{5(x+3)-(x+5)(x+2)}{(x-1)(x+2)(x+3)}.$$

Simplify, being extremely careful with signs:

$$\frac{-x^2-2x+5}{(x-1)(x+2)(x+3)}.$$

The numerator is prime (there are only two possible ways to try factoring, neither of which works).