

Instructions: Read carefully. Show your work.

1. Factor completely.

(a) (3 pts) $x^2 - 7x - 30$
 $(x - 10)(x + 3)$

(b) (4 pts) $2w^3 - 12w^2 + 18w$
First factor out the GCD: $2w(w^2 - 6w + 9)$.
Now factor the rest, noting that we have a special form: $2w(w - 3)^2$.

(c) (4 pts) $x^6 + 7x^3 - 8$
We have higher powers of x here, so we make the substitution $y = x^3$ (since x^3 is the lowest-power x term appearing). This gives:

$$y^2 + 7y - 8$$
$$(y + 8)(y - 1)$$

Undoing the substitution, we get

$$(x^3 + 8)(x^3 - 1).$$

Now both of these terms are again special forms, so we factor some more:

$$(x + 2)(x^2 - 2x + 4)(x - 1)(x^2 + x + 1).$$

Now we're done – the remaining factors are all prime.

(d) (4 pts) $x^3 + x^2 - 9x - 9$
There are 4 terms, so our only choice is to factor by grouping.

$$x^2(x + 1) - 9(x + 1)$$
$$(x^2 - 9)(x + 1)$$

Again, we note that we can still factor some more:

$$(x - 3)(x + 3)(x + 1).$$

All these factors are prime, so we're done.

2. (5 pts each) Solve each equation by factoring. State the solution set.

(a) $2m^2 - 9m - 5 = 0$

Using either the *ac* method or guess and check, you should get the factorization

$$(2m + 1)(m - 5) = 0.$$

This gives the equations $2m + 1 = 0$ or $m - 5 = 0$. Solving each, we find that the solution set is $\{-\frac{1}{2}, 5\}$.

(b) $x^3 - 2x^2 = 3x$

We need to get 0 on one side first:

$$x^3 - 2x^2 - 3x = 0.$$

Now factor out the GCD:

$$x(x^2 - 2x - 3) = 0.$$

Factoring the rest, we have

$$x(x - 3)(x + 1) = 0.$$

This gives the equations $x = 0$, $x - 3 = 0$, or $x + 1 = 0$. Solving each of these, the solution set is $\{-1, 0, 3\}$.

(c) $(3x - 7)(x^2 - 25) = 0$

This is already partly factored; we simply need to finish the job:

$$(3x - 7)(x - 5)(x + 5) = 0.$$

We then get the equations $3x - 7 = 0$, $x - 5 = 0$, or $x + 5 = 0$. Solving, we end up with a solution set of $\{-5, 5, \frac{7}{3}\}$.