

Instructions: Solutions follow.

1. Simplify the following expressions. (3 pts each)

(a) $\frac{10abc - 5ab + 15c}{-5}$

We can rewrite division as multiplication:

$$\begin{aligned}\frac{10abc - 5ab + 15c}{-5} &= -\frac{1}{5}(10abc - 5ab + 15c) \\ &= -2abc + ab - 3c.\end{aligned}$$

(b) $\frac{1}{3}xy - \frac{1}{3}xy(2 - z)$

Order of operations tells us multiplication comes first (since there's nothing to do in the parentheses), so we start by distributing:

$$\begin{aligned}\frac{1}{3}xy - \frac{1}{3}xy(2 - z) &= \frac{1}{3}xy - \frac{2}{3}xy + \frac{1}{3}xyz \\ &= -\frac{1}{3}xy + \frac{1}{3}xyz.\end{aligned}$$

2. Solve the following equations. State the solution set. State whether the equation is conditional, inconsistent, or an identity. (6 pts each)

(a) $7 - 7(x + 3) = -14$

Solving:

$$\begin{aligned}7 - 7x - 21 &= -14 \\ -7x - 14 &= -14 \\ -7x &= 0 \\ x &= 0.\end{aligned}$$

We have a value for x , so this equation is **conditional**. The solution set is $\{0\}$.

$$(b) \frac{z-5}{5} - \frac{z-3}{3} = -1$$

We start by clearing denominators. The least common denominator of 3 and 5 is 15, so we multiply both sides by 15, then solve:

$$\begin{aligned} 15 \cdot \left(\frac{z-5}{5} - \frac{z-3}{3} \right) &= (-1) \cdot 15 \\ 3(z-5) - 5(z-3) &= -15 \\ 3z - 15 - 5z + 15 &= -15 \\ -2z &= -15 \\ z &= \frac{15}{2}. \end{aligned}$$

Again, this is **conditional**. The solution set is $\{\frac{15}{2}\}$.

$$(c) -1 + 5(2x - 3) = 16x - 2(3x + 8)$$

Solving:

$$\begin{aligned} -1 + 10x - 15 &= 16x - 6x - 16 \\ 10x - 16 &= 10x - 16. \end{aligned}$$

Both sides are identical, so we don't need to do any more work. This is an **identity**, which means the solution set is \mathbb{R} (*not* $\{\mathbb{R}\}$).

$$(d) \frac{x-1}{2} + \frac{4-3x}{6} = \frac{1}{3}$$

Again, we want to clear denominators to make this easier. The common denominator is 6:

$$\begin{aligned} 6 \cdot \left(\frac{x-1}{2} + \frac{4-3x}{6} \right) &= \left(\frac{1}{3} \right) \cdot 6 \\ 3(x-1) + (4-3x) &= 2 \\ 3x - 3 + 4 - 3x &= 2 \\ 1 &= 2. \end{aligned}$$

This is clearly a false statement, so the equation is **inconsistent**. The solution set is \emptyset , or $\{\}$ (*not* $\{\emptyset\}$ or $\{0\}$).