# A Review of *Mathematica 5.0*

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## 1 Introduction

*Mathematica 5.0* is the latest version of the mathematical software package *Mathematica* published by Wolfram Research (WR). Released in June 2003, on the fifteenth anniversary of the release its first version, *Mathematica 5.0* is the first full-point upgrade in four years. Among many other upgrades and additions, version 5.0 includes optimized routines for numerical linear algebra, improved numerical solvers for ordinary and partial differential equations, support for linear differential-algebraic equations, support for fast sparse-matrix operations, and new algorithms for solving inequalities symbolically.

This review assumes some knowledge of *Mathematica* syntax and is not meant to provide an introduction to *Mathematica*. Those looking for an introduction should consult *Mathematica*'s built in tutorial (located under the help menu). For more information, see one of the following books: Abell and Braselton [1] provide a general introduction, Gray [2] provides a more advanced look, and although the book was written for a previous version of *Mathematica*, Packell [3] provides a good introduction to the use of *Mathematica* in a university-level classroom.

As *Mathematica* is a very large software package with a wide range of abilities, it is impossible to include a complete review of the new version in this document. Instead, this review will focus on some of the most significant improvements included in version 5.0. This review is divided into three sections. Section 2 focuses on the additions made to the interface/front-end, Section 3 focuses on the technical upgrades and additions, and Section 4 includes a conclusion and some suggestions for improvement in future versions of *Mathematica*.

All simulations referenced were conducted on a PC with a 700Mhz Pentium III processor and 256Mb of RAM running Windows 2000 Professional and a Macintosh G4 with dual 450Mhz processors and 256Mb of RAM running OSX.

## 2 Interface/Front-End

*Mathematica 5.0* retains the document-centered cell format of its predecessors, allowing for the evaluation of a single command or multiple commands in succession. Most common special characters (Greek letters, mathematical symbols, operators, etc.) can be added to a *Mathematica* document (notebook) by using short-cut keys and/or point-and-click palettes. Titles, section headings, and text can be easily integrated with calculations, data, plots and figures inside of a notebook in order to create a presentation-worthy mathematical document.

Version 5.0 includes support for more than fifty different file types including a variety of sparse-matrix, data, and sound formats. Version 5.0 supports the import and export of over twenty different image file types including PNG, DICOM, PDF and EPS formats. *Mathematica 5.0* notebooks can be saved as TeX, HTML or XML documents. When saving a notebook as a TeX document, *Mathematica* creates a master .tex file and an .eps file for each graphic. When saving as HTML, *Mathematica* creates a master .html file and .gif files for all graphics and most non-text input. Although the HTML output looks very professional, it cannot be edited directly due to the large number of graphics files involved. In order to make any changes to an HTML document created in *Mathematica*, the original notebook must be edited and resaved.

In addition to updated versions of *Mathematica*'s connections to Java and C/C++, version 5.0 includes a fully-functional preview version of *.NET/Link*. This toolkit allows developers to integrate *Mathematica* into applications using Microsoft's .NET framework.

Version 5.0 includes a new palette that can be used to create slideshows similar in form to those created by Microsoft's Powerpoint or the Prosper LaTeX package. The default template allows mathematical expressions, calculations, output, text, plots and movies to be easily intermixed on a slide. Personalized templates can be created as well. One feature not available with other slideshow-creating products is that *Mathematica* slideshows are "active" documents, allowing new commands to be evaluated during a presentation.

The help menu has the same format as in previous versions and remains easy to use. WR has added "advanced documentation" for a handful of commands. This new documentation includes an introduction to the command, a list of the types of problems the command can be used to solve and the details of how the command operates. For example, the advanced documentation for the `NDSolve` command is so detailed that it alone is over 150 pages long and contains 117 references.

## 3 Technical

Linear algebra, both symbolic and numeric, is core to many areas of mathematics and scientific computation. Earlier versions of *Mathematica* are known for their abilities in symbolic computation, but not for the speed of their numerical routines. WR has put significant effort into improving *Mathematica*'s numerical routines in hopes that version 5.0 can compete directly with Matlab and dedicated numerical linear algebra packages.

The numerical linear algebra routines in version 5.0 are based upon BLAS and LAPACK and are faster than the routines in previous versions. Dense 100-by-100 random element matrices are inverted and multiplied more than twice as fast as they are in version 4.2. As the size of the matrix increases, so does the factor of improvement in speed.

Previous versions of *Mathematica* treat all matrices as if they are dense. However, version 5.0 recognizes sparse matrices and uses fast sparse-matrix-specific algorithms to handle them. This leads to orders of magnitude improvement in speed for sparse-matrix calculations.

The input of sparse matrices is especially easy in version 5.0. The command

```
SparseArray[{{i_,i_}->1,{i_,j_}/;Abs[i-j]==1->-2},{5,5}]
```

creates the 5-by-5 second-order second derivative finite difference matrix

$$\begin{pmatrix} -2 & 1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 1 & -2 \end{pmatrix}.$$

For large matrices, the `SparseArray` command is orders of magnitude faster than the double do-loop command required by previous versions of *Mathematica*.

The `NDSolve` command, *Mathematica*'s numerical ordinary and partial differential equation solver, has been completely rewritten for version 5.0. A number of internal algorithms have been added including ones that can handle differential-algebraic equations and $n$-dimensional PDEs. Many of the routines existing in previous versions were improved by more efficient implementations in version 5.0.

The Kortweg-deVries equation (KdV),

$$u_t + uu_x + u_{xxx} = 0,$$

with periodic boundary conditions on $x \in [0, 40]$ and initial condition

$$u(x,0) = 9\,\mathrm{sech}^2\Big(\frac{\sqrt{3}}{2}(x-5)\Big) + \frac{9}{2}\,\mathrm{sech}^2\Big(\frac{\sqrt{3}}{2\sqrt{2}}(x-15)\Big),$$

can be solved numerically by the command

```
NDSolve[{D[u[x,t],t]+u[x,t]D[u[x,t],x]+D[u[x,t],{x,3}]==0,u[x,0]==IC[x],
u[0,t]==u[40,t]},u,{t,0,10},{x,0,40},Method->StiffnessSwitching]
```

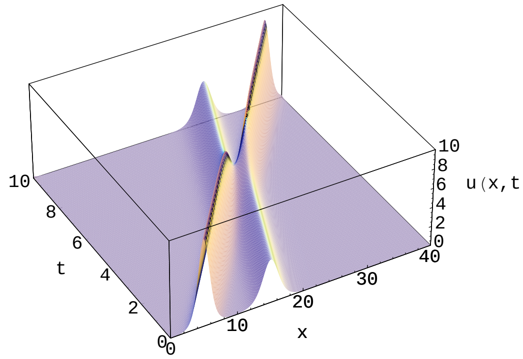


Figure 1: A numerical solution of KdV demonstrating the interaction of typical solitons and *Mathematica*'s superior graphical output.

In this command, `IC[x]` is the periodic generalization of the given initial condition. The results of this command represent a large improvement over previous versions where it is difficult to find such numerical solutions. A plot of the solution is included in Figure 1. This plot demonstrates the interaction of typical solitons in KdV and the superior graphical output of *Mathematica*.

The `DSolve` command now supports systems of differential-algebraic equations. For example, the system

$$\begin{aligned} x' &= w - y, \\ y' &= y - 2x, \\ z &= y - x, \\ w &= z - x, \\ x(0) &= 1, \\ w(0) &= 2, \end{aligned}$$

is solved by the command

```
DSolve[{x'[t]==w[t]-y[t],y'[t]==y[t]-2x[t],z[t]==y[t]-x[t],
w[t]==z[t]-x[t],x[0]==1,w[0]==2},{w[t],x[t],y[t],z[t]},t]
```

*Mathematica* returns the general solution of the system,

$$\{\{w[t] \to \tfrac{2}{3}e^{-2t}(-2+5e^{3t}),\ \ x[t] \to e^{-2t},\ \ y[t] = \tfrac{2}{3}e^{-2t}(1+5e^{3t}),\ \ z[t] = \tfrac{1}{3}e^{-2t}(-1+10e^{3t})\}\}$$

Previous versions of *Mathematica* are not able to solve or even simplify inequalities. In the new version, the `Reduce` command has been expanded in order to find solutions of mixed systems of equations and inequalities. For example, the mixed system

$$\begin{aligned} x^2 + y^2 &< 1 \\ x^2 + 2y^2 + z^2 &\geq 4 \end{aligned}$$

is solved by the command

```
Reduce[{x^2+y^2<1,x^2+2y^2+z^2>=4}]
```

*Mathematica* returns the solution in simplified form,

$$\{\{-1 < y \leq 1 \ \ \&\&\ \ -\sqrt{1-y^2} < x < \sqrt{1-y^2}\ \ \&\&\ \ (z \leq -\sqrt{4-x^2-2y^2}\ \ ||\ \ z \geq \sqrt{4-x^2-2y^2})\}\}$$

A new command named `RSolve` can be used to find solutions of difference (recurrence) equations. For example, the equation

$$x_{n+1} = ax_n + bx_{n-1},$$

where $a$ and $b$ are constants is solved by the command

```
RSolve[x[n+1]==a*x[n]+b*x[n-1],x[n],n]
```

*Mathematica* returns the general solution

$$\{\{x[n] \to 2^{-n}(a-\sqrt{a^2+4b})^n C[1] + 2^{-n}(a+\sqrt{a^2+4b})^n C[2]\}\}$$

## 4 Conclusions

*Mathematica 5.0* is a significant improvement over previous versions. However, there are some things that I would like to see changed/improved.

- It is difficult to format long sections of text inside a notebook without using a cell for each paragraph.
- There is no "print preview."
- Typos and unmatched parentheses cause a pause. This prevents rapid correction.
- Advanced documentation needs to be included for more commands. The level of detail needs to be similar to that in the advanced documentation for the `NDSolve` command.
- *Mathematica* should be able to rotate three-dimensional plots in real time by moving the mouse.
- The numerical linear algebra routines need to be faster. The routines in Matlab are faster.
- Large integers should be factored more rapidly. Maple 9 can factor some faster.
- Expressions involving Jacobi elliptic functions are not simplified using the standard identities.
- A single equals sign should not act as an assigning operator inside `DSolve` commands.

Despite these relatively minor problems, version 5.0 is a significant improvement over previous versions. *Mathematica 5.0*'s support of sparse matrices, differential-algebraic equations, difference equations, inequalities and improved speed make the upgrade worthy of serious consideration.

The suggested retail price for *Mathematica 5.0*, including one year of Premier Service, is $1880. The academic and student versions list for $895 and $139.95 respectively. Both these versions have the same functionality as the professional version. *Mathematica 5.0* can be purchased directly at www.wolfram.com.

## References


[1] Martha L. Abell and James P. Braselton. *Mathematica by Example, Third Edition*. Academic Press, 2003.

[2] John W. Gray. *Mastering Mathematica: Programming Methods and Applications*. Academic Press, 2003.

[3] Ed Packell. *Mathematica for Mathematics Teachers: Notes from an Introductory Course*. Front Range Press, 1996.