Tensor decompositions of II_1 factors arising from AFP Groups

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von Neumann Algebras

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- If $t > 0$ then $M^t := pMp$ where $p \in \mathcal{P}(M_n(\mathbb{C}) \bar{\otimes} M)$ with $(\tau_n \otimes \tau)(p) = t/n$ for n "large enough."

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$$\begin{aligned}
\mathcal{R} :=& \overline{\bigotimes_{n\in\mathbb{N}} (M_2(\mathbb{C}), \tau_2)}^{WOT} \\
=& \overline{\bigotimes_{n\in\mathbb{N}} (M_k(\mathbb{C}), \tau_k)}^{WOT} \qquad \forall\, k \in \mathbb{N} \setminus \{1\} \\
=& \overline{\bigotimes_{n\in\mathbb{N}} (M_{k_n}(\mathbb{C}), \tau_{k_n})}^{WOT} \qquad \forall\, \{k_n\} \in (\mathbb{N} \setminus \{1\})^{\mathbb{N}}
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- For $\Sigma \leqslant \Gamma$, the *virtual centralizer of* Σ *inside* $\Gamma$is

$$VC_{\Gamma}(\Sigma) = \left\{\gamma \in \Gamma : |\gamma^{\Sigma}| < \infty\right\} \leqslant \Gamma$$

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Theorem (Ozawa, Popa 03)

Let $L(\Gamma_1 \times \cdots \times \Gamma_m) \cong M \cong P_1 \bar{\otimes} \cdots \bar{\otimes} P_n$ *with* Γ_i *hyperbolic icc and* P_j *prime. Then*

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Can we remove the assumptions on Λ?

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Let $L(\Gamma_1 \times \cdots \times \Gamma_m) \cong L(\Lambda)$ *with* Γ_i *hyperbolic icc. Then* $\Lambda = \Lambda_1 \times \cdots \times \Lambda_m$ *with*

- $L(\Gamma_i) = uL(\Lambda_i)^{t_i} u^*$ *for some* $u \in \mathcal{U}(L(\Lambda))$, $t_i > 0$, *s.t.* $t_1 \cdots t_n = 1$.

Strategy of Proof:

- "Discretize" commutation to embed $L(\Gamma_1) \preceq L(\Sigma)$ in the sense of Popa for some $\Sigma < \Lambda$ with non-amenable centralizer - simultaneously analyze all embeddings $L(\Gamma_1) \preceq L(\Omega)\ \forall\ \Omega \leqslant \Lambda$ via comultiplication map and ultrapower techniques (Ioana 11).

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- Σ and $VC_\Lambda(\Sigma) = \{\lambda \in \Lambda : |\lambda^\Sigma| < \infty\}$ commensurate Λ.
- Pass to groups of finite index to recover direct product.
- Ozawa-Popa argument to build unitary and projections as required.

Consequences of Product Rigidity

Theorem (Chifan-dS-SInclair 15)

Let $L(\Gamma_1 \times \cdots \times \Gamma_m) \cong P_1 \bar{\otimes} \cdots \bar{\otimes} P_m \cong L(\Lambda)$ *with* Γ_i *hyperbolic icc. Then* $\Lambda = \Lambda_1 \times \cdots \times \Lambda_m$ *with*

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$$L(\mathbb{F}_2 \times \mathbb{F}_9) \cong L(\mathbb{F}_2)^{1/2} \otimes L(\mathbb{F}_9)^2 \cong L(\mathbb{F}_5) \otimes L(\mathbb{F}_3) \cong L(\mathbb{F}_5 \times \mathbb{F}_3)$$

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- $L(\Lambda)$ non-prime implies $\Lambda = \Lambda_1 \times \Lambda_2$.
- Classification of tensor decompositions of $L(\Lambda)$.

Other Instances of Product Rigidity

Theorem (Drimbe-Hoff-Ioana 16)

Let $\Lambda \leqslant G_1 \times \cdots \times G_n$ an icc lattice where G_i rank 1 non-compact simple Lie groups. If $L(\Lambda) = P_1 \bar{\otimes} P_2$, then $\Lambda = \Lambda_1 \times \Lambda_2$ with $L(\Lambda_i) \cong P_i^{t_i}$, $t_1 t_2 = 1$.

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Theorem (dS-Pant 17)

Let Γ be an i.c.c. group such that

$$\Gamma = \Gamma_n \rhd \Gamma_{n-1} \rhd \cdots \rhd \Gamma_1 \rhd \Gamma_0 \{e\}$$

with Γ_i / Γ_{i-1} hyperbolic i.c.c. If $L(\Gamma) \cong P_1 \bar{\otimes} P_2$, then $\Gamma = \Gamma_1 \times \Gamma_2$ with $L(\Gamma_i) \cong P_1^{t_i}$.

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Theorem (Chifan-Udrea 18)

If $\Gamma = \bigoplus_{i=1}^{\infty} \Gamma_i$ where Γ_i are icc hyperbolic property (T) groups, and $L(\Gamma) \cong L(\Lambda)$, then $\Lambda = \bigoplus_{i=1}^{\infty} \Lambda_i \oplus A$, with Λ_i property (T) and A amenable i.c.c.

Product Rigidity for AFP Groups

Theorem (Chifan-dS-Sucpikarnon 17)

Let $\Gamma = \Gamma_1 *_\Sigma \Gamma_2$ *where*

1. Γ_i, *are icc,*
2. $[\Gamma_1 : \Sigma] \geq 2$ *and* $[\Gamma_2 : \Sigma] \geq 3$,
3. $L(\Sigma)$ *is solid, e.g. hyperbolic icc.*

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Prime von Neumann Algebras from Simple Groups

Corollary

Let Γ be a group in one of the following classes:

a) *the integral two-dimensional Cremona group $Aut_k(k[x, y])$;*

b) *the Higman group* $\langle a, b, c, d | a^b = a^2, b^c = b^2, c^d = c^2, d^a = d^2 \rangle$;

c) *the Burger-Mozes groups* $\mathbb{F}_n *_{\mathbb{F}_k} \mathbb{F}_m$;

d) $PSL_2(k[t])$.

$L(\Gamma)$ *is is indefeasible as a tensor product.*

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All are AFP groups as required and are simple or (acylindrically) hyperbolic. □

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First known examples where simplicity of Γ implies structural properties for $L(\Gamma)$.

Proof of Product Rigidity

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Proof.

Note:

If $\Gamma = \Gamma_1 *_\Sigma \Gamma_2 = \Omega \times \Theta$ then

- $\Gamma_i = \Lambda_i \times \Omega$,
- $\Sigma = \Sigma_0 \times \Omega$,
- $\Theta = \Lambda_1 *_{\Sigma_0} \Lambda_2$

□

AFP Product Rigidity (Cont)

Need to show $L(\Gamma_1 *_\Sigma \Gamma_2) \cong P_1 \bar{\otimes} P_2$ implies $\Gamma = \Omega \times \Theta$.

- WLOG $P_1 \preceq L(\Sigma)$ up to finite index (Ioana 12, Vaes 12).

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- WLOG $P_1 \preceq L(\Sigma)$ up to finite index (Ioana 12, Vaes 12).
- Identify P_2 with $VC_\Gamma(\Sigma)$.
- Run previous product rigidity arguments to induce direct product.

Graph Products

Corollary

Let $\Gamma = \mathcal{G}(\Gamma_v, v \in \mathcal{V})$ *be a graph product of icc hyperbolic groups. If* $L(\Gamma) = P_1 \bar{\otimes} P_2$*, then* $\mathcal{V} = \mathcal{V}_1 \sqcup \mathcal{V}_2$, $\Gamma = \Gamma_1 \times \Gamma_2$*, where* $\Gamma_i = \mathcal{G}(\Gamma_v, v \in \mathcal{V}_i)$.

L^2 Betti numbers

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Deformation/rigidity "imports" group-theoretic aspects into the algebra.

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If Γ is an icc group with $\beta_1^{(2)}(\Gamma) \neq 0$, then $L(\Gamma) \neq P_1 \vee P_2$ whenever $P_1 \subset L(\Gamma)$ with P_i are irreducible, have prop (T), and $P_1 \cap P_2$ diffuse.

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Analysis of s-malleable deformation, $L(\Gamma) - L(\Gamma)$ bimodules, and

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Analysis of s-malleable deformation, $L(\Gamma) - L(\Gamma)$ bimodules, and pineapples (soles/Pinscer factors/poles of rigidity).

Thanks!