Monic representations for higher-rank graphs

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joint with C. Farsi, P. Jorgensen, S. Kang, J. Packer

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(Recent) History and Motivation

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(2017/18) Farsi–G–Jorgensen–Kang–Packer characterize all monic representations of $C^*(\Lambda)$ in terms of Λ-SBFS.
In particular, since Cuntz–Krieger algebras are higher-rank graph algebras, this answers the question posed by Bezuglyi–Jorgensen.

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- Monic representations; main Theorem.

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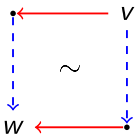

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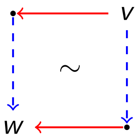
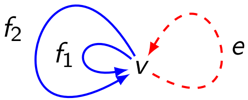


Introduced by Kumjian & Pask in 2000 to give examples of combinatorial, computable C^*-algebras, more general than $C^*(E)$.

Paths in $E \rightsquigarrow k$-dimensional rectangles in Λ.

Example and notation



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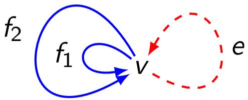

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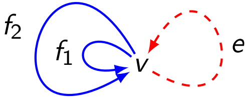

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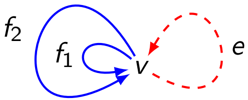

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Note that Λ^0 is the vertices of Λ.

Higher-rank graph C^*-algebras

Given a higher-rank graph Λ, $C^*(\Lambda)$ is the universal C^*-algebra generated by a family of projections $\{p_v : v \in \Lambda^0\}$ and partial isometries $\{s_\lambda : \lambda \in \Lambda\}$ satisfying the Cuntz–Krieger relations:

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(CK4) For any $v \in \text{Obj}\,\Lambda$ and any $n \in \mathbb{N}^k$,

$$p_v = \sum_{\lambda \in v\Lambda^n} s_\lambda s_\lambda^*.$$

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Moreover, $s_e s_{f_1} = s_{f_2} s_e$, so the action of $\mathbb{Z}$ on $\mathcal{O}_2 = C^*(s_{f_1}, s_{f_2})$ is given by
$$\alpha(s_{f_i}) = s_{f_{i+1}}.$$

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- Measurable maps

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- Various rules (analogous to (CK1)-(CK4)) governing how these maps and sets interact.
- Positive a.e. Radon–Nikodym derivatives

$$\Phi_\lambda := \frac{d(\mu \circ \tau_\lambda)}{d\mu}.$$

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In this setting, R_{f_1} is the upper left triangle and R_{f_2} is the lower right triangle.

This latter example is the only one I know of that has a non-constant Radon-Nikodym derivative.

Representations from Λ-SBFS

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Any Λ-semibranching function system on (X, μ) gives rise to a representation π of $C^*(\Lambda)$ on $L^2(X, \mu)$. The representation is faithful iff Λ is aperiodic.

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Λ-projective systems allow for a more general scaling function.

Λ-projective representations

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Theorem (Farsi-G-Jorgensen-Kang-Packer 2017)

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Then the operators $\{T_\lambda\}_{\lambda \in \Lambda}$ give a representation of $C^*(\Lambda)$ on $L^2(X, \mu)$:

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  This gives the Λ-SBFS representation: $T_\lambda = \pi(s_\lambda)$.

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Corollary

For Cuntz and Cuntz–Krieger algebras, Λ-projective systems and monic systems are the same thing.

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The sets $Z(\lambda)$, for $\lambda \in \Lambda$, form a compact open basis for the topology on Λ^∞:

$$Z(\lambda) = \{x \in \Lambda^\infty : x = \lambda y \text{ for some } y \in \Lambda^\infty\}.$$

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Theorem (Farsi-G-Kang-Packer, 2015)

Any finite, strongly connected k-graph admits a Λ-SBFS on the measure space (Λ^∞, M):

$$D_v = Z(v), \quad R_\lambda = Z(\lambda), \quad \tau_\lambda(x) = \lambda x.$$

Infinite paths and Λ-SBFS

When Λ is finite and strongly connected, we have a canonical measure M on Λ^∞, due to an Huef, Laca, Raeburn, and Sims.

Theorem (Farsi-G-Kang-Packer, 2015)

Any finite, strongly connected k-graph admits a Λ-SBFS on the measure space (Λ^∞, M):

$$D_v = Z(v), \quad R_\lambda = Z(\lambda), \quad \tau_\lambda(x) = \lambda x.$$

$$\Phi_\lambda = \frac{M(Z(\lambda))}{M(Z(s(\lambda))}\chi_{Z(s(\lambda))}.$$

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Theorem (Farsi-G-Jorgensen-Kang-Packer, 2017)

Let Λ be a finite k-graph with no sources. A representation ϕ of $C^*(\Lambda)$ is monic iff ϕ is unitarily equivalent to a Λ-projective representation on (Λ^∞, μ) where μ is a Borel measure on Λ^∞.

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Corollary

Any Λ-SBFS on Λ^∞ gives a monic representation. Perhaps more than one.

Checking monicity

Theorem (Farsi-G-Jorgensen-Kang-Packer)

Let Λ be a finite k-graph with no sources. Suppose we have a Λ-SBFS on (X, μ). The associated Λ-semibranching representation $\pi : C^*(\Lambda) \to B(L^2(X, \mu))$ is monic iff

$$\{S \subseteq X : \mu(S \Delta R_\lambda) = 0 \text{ for some } \lambda \in \Lambda\}$$

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For Cuntz–Krieger algebras, the monic systems which give rise to monic representations are precisely the ones associated to SBFS where the range sets R_i generate the σ-algebra.

The end

Thanks for your attention!