

West Coast Operator Algebra Seminar
Seattle University, October 6-7, 2018
Titles and Abstracts

“BROWN-DOUGLAS-FILLMORE THEORY FOR OPERATORS ON ℓ^p ”

March Boedihardjo, University of California, Los Angeles

I will define appropriate notions for the study of the Brown-Douglas-Fillmore theory for operators on ℓ^p . Some results fail in the context of ℓ^p . However, after making suitable modifications, some results become true in the context of ℓ^p . This includes that EXT is a group and homotopy invariance of EXT. Examples will be given and a version of the Brown-Douglas-Fillmore theorem for operators on ℓ^p will be conjectured.

**“MINIMAL DYNAMICAL SYSTEMS AND GROUPOIDS WITH PRESCRIBED
K-THEORY”**

Robin Deeley, University of Colorado

I will speak about joint work in progress with Ian Putnam and Karen Strung. The goal of the project is to study the existence of minimal dynamical systems and more generally minimal equivalence relations. In particular, I will discuss the following question: given a compact Hausdorff space does there exist a minimal homeomorphism on it? Although the answer is no, a similar question has a positive answer for any finite CW-complex. This question and the question of which C*-algebras can be realized as groupoid C*-algebras related to minimal dynamical systems are the motivation for our constructions.

**“GENERALIZED GAUGE ACTIONS, KMS STATES, AND HAUSDORFF
DIMENSION FOR HIGHER-RANK GRAPHS”**

Elizabeth Gillaspy, University of Montana

The infinite path space Λ^∞ of a higher-rank graph Λ is (often) a Cantor set – compact, perfect, totally disconnected. Together with Carla Farsi, Nadia Larsen, and Judith Packer, we have found several ways to put a metric on this Cantor set, and computed the associated Hausdorff dimension and measure. It turns out that the same data we needed to metrize Λ^∞ also gives us a generalized gauge action on $C^*(\Lambda)$ – and the KMS states associated to this action are intimately tied to the Hausdorff measure on Λ^∞ . To us, this was an unexpected link between the dynamical

information exhibited by a higher-rank graph (as exhibited in its KMS states) and its fractal structure.

**"A NEW PROOF OF ANALYTIC SUBORDINATION FOR FREE
CONVOLUTION, USING MONOTONE INDEPENDENCE"**

David Jekel, University of California, Los Angeles

After giving a survey of the five natural types of non-commutative independence, we present an application of monotone independence to free probability. Voiculescu showed that if X and Y are freely independent, then there exists an analytic function $F : \mathbb{H} \rightarrow \mathbb{H}$ such that $G_{X+Y} = G_X \circ F$. A less well-known proof of this result, due to Lenczewski, requires less analytical work. Using the free product Hilbert space, we argue that $X + Y = X' + Y'$ where X' has the same law as X and X' and Y' are monotone independent. It follows from monotone independence that $G_{X'+Y'} = G_{X'} \circ (1/G_{Y'})$, and thus the function $F : \mathbb{H} \rightarrow \mathbb{H}$ is given by $1/G_{Y'}$. The proof adapts to independence with amalgamation.

"BROWN'S SPECTRAL MEASURE, AND FREE BROWNIAN MOTION"

Todd Kemp, University of California, San Diego

There is no spectral theorem for non-normal operators, but in a II_1 -factor, there is a substitute for the spectral measure: Brown's measure. Defined using the Kadison-Fuglede determinant and harmonic analysis, it is a fierce object to compute and sometimes has counterintuitive properties.

In this talk, I will discuss Brown's measure, and a recent observation I made about a nice characterization of its support set, which leads to several interesting operator-theoretic questions. I will also discuss the free multiplicative Brownian motion and the current state of the art in understanding its Brown measure.

**"SPECTRAL TRIPLES, QUANTUM COMPACT METRIC SPACES, AND THE
SIERPINSKI GASKET"**

Therese Landry, University of California, Riverside

One of the fundamental tools of noncommutative geometry is Connes' spectral triple. Michel Lapidus and his collaborators have developed spectral triples for the Sierpinski gasket that recover the Hausdorff dimension, the geodesic metric, and the $\log_2 3$ -dimensional Hausdorff measure. The space of continuous, complex-valued functions on the Sierpinski gasket can be viewed as a quantum compact metric space. The

Gromov-Hausdorff distance is an important tool of Riemannian geometry, and building on the earlier work of Rieffel, Latrémolière introduced a generalization of the Gromov-Hausdorff distance to the quantum compact metric space. Aspects of geometry that can be recovered via the Gromov-Hausdorff propinquity will be discussed and compared with the geometric information that can be obtained from spectral triples. This talk will develop ideas underlying joint work in progress with Michel Lapidus.

"NON-TRACIAL GRAPH VON NEUMANN ALGEBRAS"

Brent Nelson, Vanderbilt University

Given a finite, directed, connected graph with a weighting on its edges, one can define a von Neumann algebra whose generators encode the graphical data and which is equipped with a faithful normal positive linear functional. A special case of this construction is when the weighting on the edges is induced by a weighting on the vertices, and in this case the resulting von Neumann algebra is tracial. Outside of this case, however, the von Neumann algebra is non-tracial: they are free Araki-Woods factors equipped with a free quasi-free state possibly direct sum copies of \mathbb{C} . This construction also yields an application to free products of finite dimensional algebras. This is based on joint work with Michael Hartglass.

"TENSOR DECOMPOSITIONS OF II_1 FACTORS ARISING FROM EXTENSIONS OF AMALGAMATED FREE PRODUCT GROUPS"

Rolando de Santiago, University of California, Los Angeles

We describe a family of groups whose von Neumann algebras satisfy the following rigidity phenomenon: all tensor decompositions of $L(\Gamma)$ into II_1 factors necessarily arise from direct product decompositions of the group Γ . This class includes many iterated amalgamated free products groups such as right-angled Artin groups, Burger-Mozes groups, Higman group, integral two-dimensional Cremona groups. As a consequence, we obtain several new examples of groups that give rise to prime factors.

"BOOLEAN EXTREME VALUES"

Jorge Garza Vargas, University of California, Berkeley

In the context of non-commutative probability, the spectral order allows constructing a non-commutative extreme value theory parallel to that developed decades ago

in classical probability theory. After surveying the results that were obtained in free-extreme value theory, we discuss our recent results obtained in the case of Boolean independence. Specifically, we show that the max-Boolean convolution is in some sense isomorphic to the max-classical convolution by giving a correspondence between distributions; we then use this result to obtain the Boolean max-stable laws and show that under our correspondence, the domains of attraction are preserved. This is joint work with D. V. Voiculescu.

**“COMMUTANTS MOD NORMED IDEALS AND THE MODULUS OF
QUASICENTRAL APPROXIMATION”**

Dan-Virgil Voiculescu, University of California, Berkeley

To Alain Connes’ noncommutative geometry the normed ideals of compact operators are purveyors of infinitesimals. A numerical invariant plays a key role in the study of operators modulo perturbations from these ideals. Recently new structure has appeared in these questions provided by the operator algebras which are commutants of n -tuples of selfadjoint operators modulo normed ideals. Banach space duality and K -theory aspects, as well as the recent generalization to hybrid perturbations will be discussed.

RELATIVE ENTROPY IN CONFORMAL FIELD THEORY

Feng Xu, University of California, Riverside

In this talk we will discuss motivations for relative entropies in CFT, and describe recent results based on ArXiv:1712.07283.

“SOME FREE PROBABILITY ASPECTS OF MEANDRIC SYSTEMS”

Ping Zhong, University of Wyoming

The talk will consider a family of diagrammatic objects which go under the names of meandric systems or semi-meandric systems. I will review some connections which these objects are known to have with free probability, and I will show in particular how the so-called semi-meandric polynomials can be retrieved from a natural consideration of operators on the q -Fock space. This is joint work with Alexandru Nica.